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Decomposing firm value[☆]

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What are the economic determinants of a firm's market value? We answer this question through the lens of a generalized neoclassical model of investment with quasi-fixed labor and three heterogeneous capital inputs. We estimate the structural model using firm-level data on US firms and find that, on average and depending on the industry, installed labor force accounts for 14–21% of firms' market value, physical capital accounts for 30–40%, knowledge capital accounts for 20–43%, and brand capital accounts for 6–25%. Our analysis provides direct empirical evidence for the importance of labor and intangible capital inputs for understanding firm value.

1. Introduction

Understanding the economic determinants of a firm's market value is an important question that has attracted substantial research in finance and economics. We address this question through the lens of a generalized neoclassical model of investment with four different types of quasi-fixed inputs: physical capital (machines and plants), labor (workers), and two types of intangible capital, namely knowledge capital (cumulative investment in innovation) and brand capital (cumulative investment in building brand awareness). Through structural estimation, and using data for a large cross section of publicly traded firms in the US economy, we use the model to quantify the rela-

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tive importance of the various capital inputs and labor for understanding firms' market values, both across industries and over time.

In the model, changing the quantity of the capital inputs and labor is costly, which we capture through standard adjustment cost functions. The firm's equilibrium market value depends on the shadow price and on the stock of each input, and the shadow prices can be inferred from investment and hiring data through the specification of the adjustment costs functions. When the operating profit function and the adjustment costs functions are homogeneous of degree one, the market value of each input is given by the product of the input's shadow price and its corresponding stock variable. The total market value of the firm is then the sum of the market value of all the inputs, and this additive property allows us to compute the contribution of each input for the firm's value in a straightforward manner.

To take the model to the data, we need to measure the firm-level stocks of each capital input and labor. For physical capital and labor, the data is readily available from firms' 10-K reports. For knowledge capital and brand capital, the capital stock data is not readily available given the intangible nature of these inputs. Following previous studies, we construct firm-level measures of knowledge capital stock and brand capital stock from accounting data on research and development (R&D) expenses, and advertising expenses, respectively. Accordingly, we interpret R&D expenses as a firm's investment to generate new or improve current ideas. Similarly, we interpret advertising expenses as a firm's investment to increase brand awareness. We cumulate these expenditures using the perpetual inventory method to obtain the capital stocks for knowledge capital and brand capital.

We estimate the model by minimizing the distance between the observed and the model-implied firm valuation ratios. To reduce regression noise, we estimate the model using portfolio-level moments as in Belo et al. (2013) (henceforth BXZ) and Liu et al. (2009) (henceforth LWZ).

We perform the estimation in a pooled sample that includes all firms in the economy, and also separately within different industries. Following Belo et al. (2017), we split the sample into low and high labor-skill industries (henceforth low- and high-skill industries), based on the industry-level average fraction of workers who are classified as high-skill workers in each industry. To a first approximation these industries correspond to low- and high-tech sectors of the economy.

We modify the portfolio-level estimation procedure in BXZ and LWZ in two important ways. First, we estimate the model parameters by targeting the cross-sectional portfolio-level mean rather than the portfolio-level aggregate valuation ratio. This modification avoids aggregation bias thus allowing us to recover the true firm-level structural parameters. Second, we match the realized time series of the portfolio-level valuation ratios as close as possible, as opposed to simply their time-series average. This modification is important in the context of our analysis to account for the time-varying contribution of the different inputs to firm value.

Our main empirical findings can be summarized as follows. In the pooled sample, the model performs well in explaining both the time-series and the cross-sectional variation of the valuation ratios, with a time-series R^2 of 61% and a cross-sectional R^2 of 94%. The model fit is particularly good in high-skill industries, with a time-series R^2 of 60%, whereas in the low-skill industries the model fit is more modest, with a time-series R^2 of 38%. The cross-sectional fit is good in both low- and high-skill industries, with cross-sectional R^2s above 94%. The model fit is significantly better than the fit of the standard one-physical-capital input model, especially in the time series dimension where the R^2 of the standard one-physical-capital input model is only 0% in low-skill industries and 21% in high-skill industries.

Importantly, the structural model estimates allow us to quantitatively evaluate the relative contribution of each input for firm value. In the pooled sample, and depending on how the data is aggregated for reporting purposes, we find that, on average, physical capital accounts for 22 to 30% of firms' market value, installed labor force accounts for 23 to 27%, knowledge capital accounts for 38 to 47%, and brand capital accounts for the remaining 5 to 9%. Thus, on average, the non-physical capital inputs account for the majority of firms' market value, with a share between 70 and 80%.

The relative contribution of the capital and labor inputs for firms' market value varies across industries. On average, the contribution of physical capital for firm value is higher in low-skill industries than in high-skill industries, with ranges of 40 to 43% and 21 to 30%, respectively. Related, the contribution of labor and knowledge capital for firm value increases with the average labor-skill level of the industry. In low-skill industries, the contribution of labor and knowledge capital is, on average, only 14 to 18% and 20 to 22%, respectively, whereas in high-skill industries the contribution is 21 to 24% and 43 to 51%, respectively. These results suggest that adding labor and knowledge capital to the one-physical-capital input model is especially important for understanding the valuation of firms in high-skill industries.

In addition, we find that the contribution of brand capital for firm value decreases with the average labor-skill level of the industry. Brand capital is important in low-skill industries, where it accounts on average for 17 to 25% of firm value, but it matters less in high-skill industries where it accounts on average for only 3 to 6% of firm value. Thus, our estimates show that, even though intangible capital is overall an important component of firms' market value across all industries, the type of intangible capital – knowledge or brand capital – that matters the most for firm value varies across industries. This result highlights the importance of considering heterogeneous types of intangible capital in empirical work.

The relative contribution of the capital and labor inputs for firms' market value also varies over time. In the pooled sample, the contribution of knowledge capital for firm value increased significantly from 25% in the 1970s to 45% in the 2010s, while the importance of physical capital decreased significantly from 43% in the 1970s to 23% in the 2010s. The contributions of labor and brand capital for firm value have remained relatively constant over the last four decades. The increase in the contribution of knowledge capital for firm value and the decline in the contribution of physical capital happened in both low- and high-skill industries, although the change was more pronounced in high-skill industries. This evidence suggests that the trends in the relative contribution of the inputs observed in the pooled sample cannot be simply attributed to changes in the industry composition of the US economy, but rather seems to be mainly driven by a common trend in the economy.

What explains the estimated firm value decomposition? As noted, the value of each input is determined by the product of the input's shadow price and its book value. In equilibrium, the shadow price of each input equals its marginal investment cost, which depends on investment and hiring and on the adjustment cost parameters. In particular, all else equal, the more costly it is to adjust inputs to changing economic conditions, the more valuable the existing stock of the inputs. Thus, understanding the adjustment costs estimates is important for understanding the relative contribution of each input for firm value.

Our estimates show that adjusting the four inputs in response to changing economic conditions is fairly costly, especially labor and knowledge capital. Using the estimates from the pooled sample, we find that a firm's annual labor adjustment costs represent, on average, about 6.5% of total annual sales. In addition, knowledge capital adjustment costs represent, on average, about 10% of total annual sales. These figures are significantly higher than the physical capital and brand capital average adjustment costs of about 0.9% and 0.5% of total annual sales, respectively. The adjustment costs estimates of the different capital and labor inputs vary across industries, and, except for brand capital, are higher in high-skill industries than in low-skill industries. Overall, this evidence confirms that it is rather difficult, and hence costly, to build up a highly skilled labor force and innovative knowledge base, both of which represent fairly scarce, and hence valuable, economic resources.

Taken together, our analysis shows that non-physical capital inputs are quantitatively important determinants of firms' market values, and hence provides direct empirical evidence supporting models with multiple inputs as sources of firm value.

The rest of the paper proceeds as follows. Section 2 discusses the related literature. Section 3 presents the model. Section 4 introduces the functional forms and describes the estimation procedure and the data. Section 5 presents the empirical results. Finally, Section 6 concludes. A separate appendix with additional results and robustness checks is available online.

2. Related literature

Our work is closely related to the large literature on firm valuation.[1] We extend the supply approach to valuation developed in BXZ, which builds on previous work by Barnett and Sakellaris (1999), to a setup in which multiple and heterogeneous quasi-fixed capital and labor inputs contribute to a firm's market value. Hence, our work is also related to the literature on labor demand and capital investment which investigates the importance of capital heterogeneity and capital and labor adjustment costs for understanding firms' investment and hiring dynamics.[2] We contribute to this literature by providing new structural estimates of adjustment costs for multiple types of capital and labor inputs using asset prices, which can be used to guide future research with models featuring multiple inputs.

Following the important work of Ohlson (1995), a large number of empirical studies have, similar to our approach, regressed firm valuation ratios on several firm characteristics. We differ from most of these empirical studies in that our estimates are grounded in economic theory and thus have a structural interpretation.

Hall (2001) and McGrattan and Prescott (2000) show that intangible capital matters for understanding aggregate stock market valuations. Li and Liu (2012) and Vitorino (2014) confirm the importance of intangible capital for firm-level valuation in a neoclasssical model via structural estimation. We build on their work by considering a more general model of the firm that includes heterogeneous intangible capital (knowledge and brand capital) inputs, and also quasi-fixed labor.

Eisfeldt and Papanikolaou (2013) show that firms with more organization capital, a form of intangible capital, are riskier than firms with less organization capital. Peters and Taylor (2017) incorporate organization capital into the measurement of a novel proxy for the Tobin's Q which explains total firm investment in physical and intangible capital better than standard proxies. Following Lev and Radhakrishnan (2005), the previous two papers construct a measure of organization capital from selling, general, and administrative (SG&A) expenses accounting data. This is a broad measure capturing the value of the labor force (as it accounts for the costs of training workers), knowledge capital (as it often includes R&D expenditures), and brand capital (as it accounts for advertising expenses), and which also includes other operational expenses.[3] We provide a detailed decomposition of firm value by using measures of the separate components of intangible capital instead of this broader measure. In Section 5.7 we also discuss the estimation results based on a version of the model using organization capital.

Falato et al. (2014), building on earlier work by Corrado et al. (2009) and Corrado and Hulten (2010), measure intangible capital from past R&D expenses accounting data, and show that intangible capital is the most important firm-level determinant of corporate cash holdings. We

[1] See BXZ for an overview of this literature in Finance, Economics, and Accounting.

[2] Bond and Van Reenen (2007) survey the labor demand and capital investment literature. A noncomprehensive list of studies examining the importance of capital heterogeneity include Abel (1985), Abel and Eberly, 2001, Hayashi and Inoue (1991), Chirinko (1993), and Gonçalves et al. (2020).

[3] As discussed in Peters and Taylor (2017), companies typically report SG&A, and R&D separately, but Compustat almost always adds R&D expenses to SG&A reporting them together in the variable XSGA.

provide additional evidence of the importance of intangible capital from a firm valuation perspective.

Merz and Yashiv (2007) show that labor matters for understanding the dynamics of the aggregate stock market value. Building on Cochrane (1991), they estimate a model of an aggregate representative firm facing adjustment costs in both physical capital and labor. We extend Merz and Yashiv (2007)'s setup by including two additional types of costly intangible capital, and performing the analysis at the firm-level, which allows us to exploit not only time-series data, but also firm-level cross-sectional data.

3. The model of the firm

We consider a neoclassical model of the firm as in LWZ/BXZ, extended to a setup with several quasi-fixed inputs. Time is discrete and the horizon infinite. Firms choose costlessly adjustable inputs (e.g., materials, energy) each period, while taking their prices as given, to maximize operating profits (revenues minus the expenditures on these inputs). Because we treat labor and intangible capital as quasi-fixed inputs, the labor costs and the investments in intangible capital are excluded from our definition of operating profits. Then, taking these operating profits as given, firms optimally choose the physical and intangible capital investments, hiring, and debt to maximize their market value of equity.

To save on notation, we denote a firm's i set of capital and labor input stocks at time t, as $\boldsymbol{K_{it}}$ (variables in bold represent a vector). This set includes the physical capital stock (K_{it}^{P}), the labor stock (L_{it}), the knowledge capital stock (K_{it}^{K}), and the brand capital stock (K_{it}^{B}). Similarly, we denote a firm's i set of investments in the different inputs (hiring in the case of the labor input) at time t, as $\boldsymbol{I_{it}}$. This set includes the investment in physical capital (I_{it}^{P}), the investment in labor stock, that is, gross hiring (H_{it}), the investment in knowledge capital (I_{it}^{K}), and the investment in brand capital (I_{it}^{B}).

The laws of motion of the firm's capital inputs and labor force are given by:

$$K_{it+1}^{P} = I_{it}^{P} + (1-\delta_{it}^{P})K_{it}^{P} \tag{1}$$

$$L_{it+1} = H_{it} + (1-\delta_{it}^{L})L_{it} \tag{2}$$

$$K_{it+1}^{K} = I_{it}^{K} + (1-\delta_{it}^{K})K_{it}^{K} \tag{3}$$

$$K_{it+1}^{B} = I_{it}^{B} + (1-\delta_{it}^{B})K_{it}^{B}, \tag{4}$$

where δ_{it}^{P}, δ_{it}^{K}, and δ_{it}^{B} are the exogenous depreciation rates of physical, knowledge, and brand capital, respectively. δ_{it}^{L} is the employee quit rate, i.e., the rate at which the workers leave the firm for voluntary reasons.

3.1. Technology

The operating profit function for firm i at time t is $\Pi_{it} \equiv \Pi(\boldsymbol{K_{it}}, \boldsymbol{X_{it}})$, in which $\boldsymbol{X_{it}}$ denotes a vector of exogenous aggregate and firm-specific shocks. Firms incur adjustment costs when investing and hiring. The adjustment costs function is denoted by $C_{it} \equiv C(\boldsymbol{I_{it}}, \boldsymbol{K_{it}})$. This function is increasing and convex in investment and hiring, and decreasing in the capital stocks and the labor force. For physical and intangible capital inputs the adjustment costs include, for example, planning and installation costs, and costs related with production being temporarily interrupted. For labor, these costs include the costs of hiring and firing workers and the costs of training new workers, among other costs. We assume that the firm's operating profit function and adjustment costs function are both homogeneous of degree one and we specify the functional forms in the empirical section below.

3.2. Taxable profits and firm's payouts

Firms can issue debt to finance their operations.[4] At the beginning of time t, firm i issues an amount of debt, denoted B_{it+1}, which must be repaid at the beginning of time $t+1$. The gross corporate bond return on B_{it} is denoted r_{it}^{B}.

We can write taxable corporate profits, denoted TCP, as operating profits minus intangible capital investments (which are expensed), labor costs, physical capital depreciation, adjustment costs, and interest expense:

$$TCP_{it} = \Pi_{it} - I_{it}^{K} - I_{it}^{B} - W_{it}L_{it} - \delta_{it}^{P}K_{it}^{P} - C_{it} - (r_{it}^{B}-1)B_{it}.$$

Thus, adjustment costs are expensed, consistent with treating them as foregone operating profits.

Let τ_{it} be the corporate tax rate. The firm's payout, denoted D, is then given by:[5]

$$\begin{aligned} D_{it} \equiv\ & (1-\tau_t)[\Pi_{it} - C_{it} - I_{it}^{K} - I_{it}^{B} - W_{it}L_{it}] - I_{it}^{P} + B_{it+1} \\ & -r_{it}^{B}B_{it} + \tau_t\delta_{it}^{P}K_{it}^{P} + \tau_t(r_{it}^{B}-1)B_{it}, \end{aligned} \tag{5}$$

in which $\tau_t\delta_{it}^{P}K_{it}^{P}$ is the depreciation tax shield, and $\tau_t(r_{it}^{B}-1)B_{it}$ is the interest tax shield.

3.3. Equity value

Firm i takes the stochastic discount factor, denoted $M_{t+\triangle t}$, from period t to $\triangle t$ as given when maximizing its cum-dividend market value of equity:

$$V_{it} \equiv \max_{\{\mathbf{I}_{it+\triangle t}, \mathbf{K}_{it+\triangle t+1}, B_{it+\triangle t+1}\}_{\triangle t=0}^{\infty}} E_t\left[\sum_{\triangle t=0}^{\infty} M_{t+\triangle t}D_{it+\triangle t}\right], \tag{6}$$

subject to a transversality condition given by $\lim_{T\to\infty} E_t[M_{t+T}B_{it+T+1}] = 0$, and the laws of motion for the capital inputs and labor given by Eqs. (1) to (4).

Let $P_{it} \equiv V_{it} - D_{it}$ be the ex-dividend equity value. In Appendix A we show that, given the homogeneity of degree one of the operating profit and adjustment costs functions, the firm's value maximization implies that:

$$P_{it} + B_{it+1} = q_{it}^{P}K_{it+1}^{P} + q_{it}^{L}L_{it+1} + q_{it}^{K}K_{it+1}^{K} + q_{it}^{B}K_{it+1}^{B}, \tag{7}$$

[4] We include debt in the model to better fit the data, but other than that, for parsimonious reasons, we keep the financing side of the firm as simple as possible.

[5] Note that physical capital investment and intangible capital investments are treated differently given the different accounting rules. Investment in physical capital is spread out over time and expensed as depreciation, while the intangible capital costs (which in our case are R&D and advertising expenses) are mostly treated as expenses at the time that they occur.

in which

$$q^P_{it} \equiv 1 + (1-\tau_t)\partial C_{it}/\partial I^P_{it} \tag{8}$$

$$q^L_{it} \equiv (1-\tau_t)\partial C_{it}/\partial H_{it} \tag{9}$$

$$q^K_{it} \equiv (1-\tau_t)\left[1 + \partial C_{it}/\partial I^K_{it}\right] \tag{10}$$

$$q^B_{it} \equiv (1-\tau_t)\left[1 + \partial C_{it}/\partial I^B_{it}\right], \tag{11}$$

and $\partial C_{it}/\partial x$ denotes the first derivative of the adjustment costs function with respect to variable x, and q^P_{it}, q^L_{it}, q^K_{it} and q^B_{it} measure the shadow prices of physical capital, labor, knowledge capital, and brand capital, respectively (i.e., the Lagrange multipliers of Eqs. (1) to (4)). The valuation Eq. (7) is simply an extension of Hayashi (1982)'s result to a multi-factor inputs setting.

According to Eq. (7) the firm's market value is given by the sum of the value of the firm's installed labor and capital inputs. This additive feature allows us to compute the fraction of firm value that is attributed to each input (henceforth referred to simply as "input-shares") in a straightforward manner as follows:

$$\mu^P_{it} = \frac{q^P_{it}K^P_{it+1}}{q^P_{it}K^P_{it+1} + q^L_{it}L_{it+1} + q^K_{it}K^K_{it+1} + q^B_{it}K^B_{it+1}} \tag{12}$$

$$\mu^L_{it} = \frac{q^L_{it}L_{it+1}}{q^P_{it}K^P_{it+1} + q^L_{it}L_{it+1} + q^K_{it}K^K_{it+1} + q^B_{it}K^B_{it+1}} \tag{13}$$

$$\mu^K_{it} = \frac{q^K_{it}K^K_{it+1}}{q^P_{it}K^P_{it+1} + q^L_{it}L_{it+1} + q^K_{it}K^K_{it+1} + q^B_{it}K^B_{it+1}} \tag{14}$$

$$\mu^B_{it} = \frac{q^B_{it}K^B_{it+1}}{q^P_{it}K^P_{it+1} + q^L_{it}L_{it+1} + q^K_{it}K^K_{it+1} + q^B_{it}K^B_{it+1}}. \tag{15}$$

The fundamental goal of the empirical analysis is to characterize these input-shares, including their variation across industries and over time.

3.4. Discussion of key model assumptions

Our assumption that the firm's operating profit function and adjustment costs function are both homogeneous of degree one allows us to obtain a closed-form expression for the firm's equilibrium market value given in Eq. (7), which depends on the model parameters and on firm-level data.[6] This greatly simplifies the empirical analysis because it allows us to estimate the structural model directly using observed data, as opposed to indirectly through data simulated from the model (for example, by using the simulated method of moments).

The assumption of homogeneity of degree one of the operating profit function does not necessarily imply that the goods market is perfectly competitive, nor that the firm's production function exhibits constant returns to scale. In the online appendix, we consider, as an example, the maximization problem of a firm with a production function that exhibits decreasing returns to scale in physical capital, and that faces a downward sloping demand curve (that is, the firm has market power). The firm's intangible capital does not enter its production function as an input but rather it affects consumer demand, since a higher stock of intangible capital, for example brand capital, can increase consumers' willingness to pay for the firm's goods. We show that, in this setup, there exists a set of parameter values for the demand and production functions in which the resulting operating profit function is homogeneous of degree one in the two capital inputs (physical and intangible capital).

The previous example further shows that knowledge or brand capital may matter for firm value not necessarily through their effect on production (as in the case of physical capital and labor), but through their effect on consumers' willingness to pay, and hence on firms' operating profits. Indeed, our model specification with a linearly homogeneous operating profit function in the four inputs can be also interpreted as a model of the firm in which the production function has decreasing returns to scale in physical capital and labor, and the firm has some market power, potentially driven by the firm's intangible (knowledge and brand) capital.

4. Estimation methodology and data

In this section we specify the functional forms and describe the estimation procedure. In addition, we describe the data, including the measurement of the intangible capital stocks, and report descriptive statistics of the key variables used in the analysis.

4.1. Functional forms

The valuation Eq. (7) only requires the specification of the adjustment costs function, not of the operating profit function. We consider the following quadratic adjustment costs function:

$$C_{it} = \frac{\theta_P}{2}\left(\frac{I^P_{it}}{K^P_{it}}\right)^2 K^P_{it} + \frac{\theta_L}{2}\left(\frac{H_{it}}{L_{it}}\right)^2 W_{it}L_{it} + \frac{\theta_K}{2}\left(\frac{I^K_{it}}{K^K_{it}}\right)^2 K^K_{it} + \frac{\theta_B}{2}\left(\frac{I^B_{it}}{K^B_{it}}\right)^2 K^B_{it}, \tag{16}$$

in which W_{it} is the wage rate (which the firm takes as given), and $\theta_P, \theta_L, \theta_K, \theta_B > 0$ are the parameters that control the magnitude of the adjustment costs of each input. Labor adjustment costs are proportional to the firm's wage bill, as in Bloom (2009). This helps to make the units of the labor adjustment costs (measured in number of workers) similar to the other capital inputs which are measured in (real) dollar values.

This functional form implies that the shadow prices of labor and the capital inputs can be inferred from firm-level data on investment, hiring, capital and labor stocks, wages,

[6] In the online appendix we estimate the operating profit function using the methods of Olley and Pakes (1996) and Ackerberg et al. (2015) and find empirical support for a linearly homogeneous specification.

and taxes, and are given by:

$$q_{it}^P \equiv 1 + (1-\tau_t)\theta_P\left(\frac{I_{it}^P}{K_{it}^P}\right) \tag{17}$$

$$q_{it}^L \equiv (1-\tau_t)\theta_L\left(\frac{H_{it}}{L_{it}}\right)W_{it} \tag{18}$$

$$q_{it}^K \equiv (1-\tau_t)\left[1 + \theta_K\left(\frac{I_{it}^K}{K_{it}^K}\right)\right] \tag{19}$$

$$q_{it}^B \equiv (1-\tau_t)\left[1 + \theta_B\left(\frac{I_{it}^B}{K_{it}^B}\right)\right]. \tag{20}$$

We adopt a simple quadratic adjustment cost specification for parsimonious reasons and to avoid parameter proliferation. There are several implicit assumptions in our simple specification that are worth discussing. First, we assume that adjustment costs depend on the gross (as opposed to net) flow of the inputs. For example, in the case of labor, firms may incur adjustment costs even if the number of workers does not change (net flow is zero) but there is labor turnover, because the firm needs to hire and train the new workers. The importance of using gross labor flows instead of net flows is consistent with the empirical evidence in Hamermesh (1995). For consistency, we adopt the same specification for all the inputs.

Second, we only consider smooth adjustment costs and thus ignore non-convex adjustment costs that lead to inaction regions and lumpy investment. According to the analysis in Section 3.3, the assumption of smooth adjustment costs allow us to derive a closed form expression for the firm's equilibrium value as a function of firm real variables and model parameters, which greatly simplifies the estimation of the model. This specification is reasonable in our context because our sample (described below) consists of publicly listed firms for which the evidence of inaction/lumpiness in investment is more limited than for establishment-level data.

Third, we assume symmetry across positive and negative input adjustments (e.g., in the case of labor, the adjustment cost of hiring or firing one worker is the same), and we also assume that the curvature parameter of the adjustment costs function is two (quadratic). In Section 5.7 below, we show that using a more flexible representation of the adjustment costs function produces a model fit that is similar to the simpler specification considered here.

4.2. Estimation procedure

The valuation Eq. (7) links firm value to the value of its labor and capital inputs. Since firm values are not necessarily stationary, it is useful to scale the variables in this equation for estimation purposes. Accordingly, we scale the variables in the equation by dividing them by the sum of the firm's capital inputs, which we denote as A_{it+1}, a measure of the firm's total (effective) assets given by $A_{it+1} \equiv K_{it+1}^P + K_{it+1}^K + K_{it+1}^B$. For scaling purposes, we do not include labor in this definition of total assets because labor is measured in different units (number of workers as opposed to dollars in real terms). Accordingly, we write a firm's valuation ratio ($VR_{it} \equiv (P_{it} + B_{it+1})/A_{it+1}$) as:

$$VR_{it} = q_{it}^P \frac{K_{it+1}^P}{A_{it+1}} + q_{it}^L \frac{L_{it+1}}{A_{it+1}} + q_{it}^K \frac{K_{it+1}^K}{A_{it+1}} + q_{it}^B \frac{K_{it+1}^B}{A_{it+1}}. \tag{21}$$

The left-hand side (LHS) of Eq. (21) can be directly measured in the data from equity price and debt data (and measures of the capital stocks, which we discuss below). The right hand side (RHS) of Eq. (21) is the predicted valuation ratio from the model, which we will denote as $\widehat{VR}_{it}$, and depends on firm-level real variables and model parameters.

Eq. (21) establishes an exact relationship between a firm's observed valuation ratio and its model-implied valuation ratio at each point in time. Using Eq. (21) and firm-level data to directly estimate the model parameters is challenging, however. First, firm-level data can be very noisy which can make estimation at the firm-level very sensitive to outliers. Second, firm-level moments are sensitive to firm entry and exit, and are likely affected by missing observations. These are important considerations in our analysis due to the length of the firm-panel studied and because the R&D and advertising expenses data needed to construct the knowledge capital and brand capital stocks are missing for a nontrivial fraction of the firms in Compustat (as discussed in Section 4.4 below).

To circumvent the previous issues while maximizing the use of the information in our sample, we estimate the model parameters using portfolio-level moments as in BXZ, which in turn follow the original approach in LWZ. The use of portfolio-level moments, a common practice in the asset pricing literature, has several attractive features in our context. First, it allows us to reduce the noise in the firm-level data, and hence obtain more accurate parameter estimates and measures of model fit such as the regression R^2. Second, portfolio-level moments are less sensitive, and hence more stable, to firm entry and exit, and to missing firm-level observations. Finally, the use of portfolio-level moments allows us to characterize the data in a more parsimonious manner as the number of portfolios is naturally smaller than the number of firms in the data.

We proceed as follows. In theory, at each point in time, any cross-sectional moment of the observed firm-level valuation ratios in the LHS of Eq. (21) should be equal to any corresponding cross-sectional moment of the model-implied firm-level valuation ratios in the RHS of Eq. (21). Accordingly, for each portfolio j and for each year t, we compute the cross-sectional mean observed and model-implied valuation ratios ($\overline{VR}_{jt}$ and $\widehat{\overline{VR}}_{jt}$, respectively) of the firms in the portfolio as follows:

$$\overline{VR}_{jt} = \sum_i \frac{VR_{it}}{N_{jt}}$$

$$\widehat{\overline{VR}}_{jt}(\Theta) = \sum_i \frac{\widehat{VR}_{it}}{N_{jt}}\ ,\ i \in \text{portfolio } j,$$

where Θ represents the vector of structural parameters, $\Theta = [\theta_P, \theta_L, \theta_K, \theta_B]$, and N_{jt} is the number of firms in portfolio j at time t. We target cross-sectional mean valuation ratios as these moments capture the economic behavior of a typical (average) firm in the economy.[7]

We then proceed under the standard assumption that the portfolio-level cross-sectional mean valuation ratios are observed with error by the econometrician:

$$\overline{VR}_{jt} = \widehat{\overline{VR}}_{jt}(\Theta) + \varepsilon_{jt}, \tag{22}$$

where ε is the error. Based on Eq. (22), we then estimate the model parameters by minimizing the sum of the squared distances between the portfolio-level observed and model-implied cross-sectional mean valuation ratio at each point in time:

$$\widehat{\Theta} = \arg\min_{\Theta} \frac{1}{TN} \sum_{t=1}^{T} \sum_{j=1}^{N} \left(\overline{VR}_{jt} - \widehat{\overline{VR}}_{jt}(\Theta) \right)^2, \tag{23}$$

where T is the number of years in the sample, and N is the number of portfolios. Finally, we compute Newey-West standard errors with lag equal to three years, to account for possible cross-sectional and time-series correlations.

An attractive feature of our estimation approach is that it corresponds to a simple linear ordinary least squares (OLS) estimation of (adjusted) portfolio-level average valuation ratios on portfolio-level averages of firm-characteristics. This is due to the linear relationship between the model-implied valuation ratio and the parameters, combined with the use of portfolio-level cross-sectional means as target moments.[8]

Our estimation procedure differs from the portfolio-level estimation procedure in BXZ and LWZ in two important ways. First, in each period t, we target the cross-sectional portfolio-level mean instead of targeting a portfolio-level aggregate valuation ratio (which aggregates each portfolio-level characteristic separately using the Fama and French (1993) approach). This modification is important to recover the firm-level structural parameters because, as we show in the online appendix, the procedure in BXZ/LWZ is subject to an aggregation bias and hence the estimates do not have a structural interpretation.[9] Naturally, the ability to recover the firm-level structural parameters is important to provide a proper decomposition of the market value of the firm.

Second, our estimation procedure requires the model to match the realized time series of the portfolio-level valuation ratios as close as possible, rather than simply their time-series average as in BXZ and LWZ. This modification is important in the context of our analysis to account for the time-varying contribution of the different inputs to firm value. At the same time, the time-series data provides relevant information for the identification of the model parameters.

4.3. Industry classification and portfolio sorts

We estimate the model in a pooled sample with all firms in the economy and also separately across two broad industries, which we refer to as low- and high-skill industries. Following Belo et al. (2017), this industry classification is constructed based on the average labor-skill level of the employees in a given narrowly-defined industry (see Appendix B for details). To a first approximation these industries correspond to low- and high-tech sectors of the economy.

This industry classification is interesting for the purposes of our analysis because the adjustment costs and the size of the intangible capital stocks vary in a systematic way across low- and high-skill industries. First, as discussed in Belo et al. (2017) (and references therein), previous empirical studies find that it is more costly to replace a high-skill worker than a low-skill worker. These relatively higher labor adjustment costs in the high-skill industries imply that, all else equal, labor should represent a higher fraction of firm value in these industries. Second, Belo et al. (2017) also provide evidence that R&D expenditures, and hence the size of knowledge capital, is relatively higher in high-skill industries which implies that, all else equal, knowledge capital should represent a higher fraction of firm value in these industries.

As noted above, the estimation is performed at the portfolio-level, which requires the specification of a sorting variable to create the portfolios. To minimize the influence of a particular choice of sorting variable on the results, we consider several sorting variables. In addition, it is useful to sort on variables that are likely to generate a large dispersion in the RHS variables in Eq. (21), to span the state space and thus improve the identification of the model parameters. Accordingly, we form four sets of portfolios sorted on the following variables: $\left(\frac{I^P_{it}}{K^P_{it}}\right)\left(\frac{K^P_{it+1}}{A_{it+1}}\right)$, $\left(\frac{H_{it}}{L_{it}}\right)\left(\frac{W_{it}L_{it+1}}{A_{it+1}}\right)$, $\left(\frac{I^K_{it}}{K^K_{it}}\right)\left(\frac{K^K_{it+1}}{A_{it+1}}\right)$, and $\left(\frac{I^B_{it}}{K^B_{it}}\right)\left(\frac{K^B_{it+1}}{A_{it+1}}\right)$.

Since these variables exhibit positive serial correlation, sorting on these variables is likely to generate a dispersion in the realized (i.e., after portfolio-formation) values of the RHS variables in Eq. (21). We then follow Fama and French (1993) in constructing the portfolios. Specifically, we sort all stocks in each year t into ten portfolios based on the deciles of the sorting variable of each firm for the fiscal year ending in $t-1$. The portfolios are re-balanced at the end of each year. This procedure gives a total of 40 portfolios.

4.4. Data

Here we provide a general description of the data and, in particular, we explain how we construct the capital

[7] Arguably, our model is less appropriate for the valuation of superstar firms, such as Apple or Facebook, which are likely to derive a large part of their market value from features not captured by our model.

[8] To show this claim more formally, define the following variables: $\overline{VR}^A_{jt} = \frac{1}{N_{jt}}\sum_{i\in j}\frac{(P_{jt}+B_{jt+1}-K^P_{jt+1}-(1-\tau_t)K^K_{jt+1}-(1-\tau_t)K^B_{jt+1})}{A_{jt+1}}$, $\overline{IPA}_{jt} = \frac{1}{N_{jt}}\sum_{i\in j}(1-\tau_t)\frac{I^P_{it}}{K^P_{it}}\frac{K^P_{it+1}}{A_{it+1}}$, $\overline{HLA}_{jt} = \frac{1}{N_{jt}}\sum_{i\in j}(1-\tau_t)\frac{H_{it}}{L_{it}}\frac{W_{it}L_{it+1}}{A_{it+1}}$, $\overline{IKA}_{jt} = \frac{1}{N_{jt}}\sum_{i\in j}(1-\tau_t)\frac{I^K_{it}}{K^K_{it}}\frac{K^K_{it+1}}{A_{it+1}}$, and $\overline{IBA}_{jt} = \frac{1}{N_{jt}}\sum_{i\in j}(1-\tau_t)\frac{I^B_{it}}{K^B_{it}}\frac{K^B_{it+1}}{A_{it+1}}$. We can then write Eq. (22) as:

$$\overline{VR}^A_{jt} = \theta_P\overline{IPA}_{jt} + \theta_L\overline{HLA}_{jt} + \theta_K\overline{IKA}_{jt} + \theta_B\overline{IBA}_{jt} + \varepsilon_{jt} \tag{24}$$

which establishes a linear relation between the portfolio-level (adjusted) valuation ratio and portfolio-level characteristics. Thus, our objective function in (23) corresponds to a simple linear OLS regression of Eq. (24).

[9] See also, Zhang 2017; Gonçalves et al. 2020; and Belo et al. 2019, for a discussion of the aggregation bias.

stocks. To save space, we report additional details about the data in Appendix B, including the construction of investment price indices, the industry-level labor data, and the treatment of missing R&D and advertising data.

Sample selection: The sample consists of US publicly traded firms from 1950 to 2016. The estimation starts in 1975 because that is the year in which the Financial Accounting Standards Board (FASB) required firms to disclose and expense all R&D expenditures (used in the construction of the knowledge capital stock) during the year in which these expenses were incurred (we use the data prior to 1975 to construct the initial intangible capital stocks as described below). The firm-level data are from the Center for Research in Security Prices (CRSP)/Compustat Merged (CCM) – Fundamentals Annual database. We limit our analysis to firms incorporated in the US (Compustat fic="USA") that trade on major stock exchanges (NYSE, AMEX, and NASDAQ) (CRSP exchange codes 1, 2, and 3), for which the native currency is US dollars (Compustat curcd="USD"), and that have information on their ordinary common shares traded (CRSP share codes 10 and 11). We exclude firms with primary standard industrial classifications (SIC) between 4900 and 4999 (regulated utilities) and between 6000 and 6999 (financial services). We drop firm-year observations with missing market values, number of employees or physical capital, and we only include firms that report R&D expenses at least once during their lifetime. Further, we drop firm-year observations with missing advertising or R&D data whenever we are not able to fill in the missing data as described in the Appendix B. The final sample used for the baseline estimation of the model includes annual data from 4,610 firms for the period from 1975 to 2016, which corresponds to 52,010 firm-year observations.

Physical capital data: The initial physical capital stock, K_{i0}^P, is given by net property, plant, and equipment (data item PPENT). The capital depreciation rate, δ_{it}^K, is the amount of depreciation (data item DP) divided by the beginning of the period capital stock. We then construct a measure of the firm's capital stock at current prices. Specifically, we construct an investment-price adjusted capital stock that accounts for changes in the real cost of physical capital investment by repricing last period's capital stock using today's price of investment (P_t^P) as $K_{t+1}^P = K_t^P(1-\delta_t)\frac{P_{t+1}^P}{P_{t+1}^P} + I_{t+1}$. Following Zhang (2017) we infer physical capital investment from the law of motion of capital by inverting the previous law of motion of physical capital equation and solving for investment (accounting for inflation). This procedure guarantees that the investment and physical capital data are consistent with the law of motion for physical capital in the model.[10]

Labor data: The labor stock, L_{it}, is the number of employees (Compustat data item EMP). The labor market data on wage rates and labor quit rates is not available at the firm level for most firms (the firm-level wage bill data in Compustat is missing for more than 80% of the firms in our sample). Thus, we measure these variables at the industry-level following the procedure described in Appendix B.

Knowledge capital data: Following Falato et al. (2014) we construct the firm's stock of knowledge capital from past expenditures data on R&D (Compustat data item XRD) and using the perpetual inventory method as follows:[11]

$$K_{t+1}^K = K_t^K(1-\delta^K)\frac{P_{t+1}^K}{P_t^K} + I_{t+1}^K, \tag{25}$$

where P_t^K is the R&D price index.[12] To implement the law of motion in Eq. (25) we must choose an initial stock and a depreciation rate. Using the perpetual inventory method, we set the initial stock to:

$$K_0^K = \frac{I_0^K}{g^K + \delta^K - \pi^K(1-\delta^K)},$$

in which I_0^K is the firm's investment in knowledge capital in the first year in the sample, and π^K is the average (net) growth rate of the price index for R&D, which is 3.2% in the sample period used for the estimation. We let g^K be industry specific and set it to be equal to the average growth rate of the R&D investments in that industry; in practice, we consider 10 industries classified according to the average labor skill level of the industry (we describe the labor skill data in Appendix B). As for the knowledge capital depreciation rate, we use the recommended depreciation rates of R&D assets based on the Bureau of Economic Analysis-National Science Foundation (BEA-NSF) data set as estimated by Li (2012) and reported for each industry in Li (2012)'s Table 4, column 3. For the industries not reported in Li (2012) we use a 15% depreciation rate following Peters and Taylor (2017). In the online appendix we show that the main results are robust to reasonable variations of the knowledge capital depreciation rates. Once we have the initial capital stock, we iterate forward using the appropriate depreciation rate, R&D expenses, and investment price index. The investment rate on knowledge capital is then given by the ratio of the current period investment and the beginning of the period corresponding knowledge capital stock I_t^K/K_t^K.

[10] Several studies (for example, LWZ) measure investment in physical capital, I_{it}^P, as capital expenditures (item CAPX) minus sales of property, plant, and equipment (item SPPE), and set SPPE to zero if missing. As shown in Zhang (2017), this procedure generates investment series that violate the assumed law of motion of physical capital for several observations. The main reason for this fact is that CAPX excludes acquisitions, that is, increases in the firm's capital stock due to the acquisition of other firms. Our measure of physical capital stock improves the fit of the baseline one-physical-capital input model thus providing a higher hurdle for our multi-capital input model.

[11] The Bureau of Economic Analysis uses a similar methodology to construct the stock of Research and Development capital, see Sliker (2007).

[12] The XRD data item in Compustat often includes not just the R&D expenses reported by companies but also the R&D acquired by companies that is deemed to not have alternative future use (data item RDIP, In-Process R&D Expense). RDIP is included in XRD whenever the item XRD_FN (XRD footnote) in Compustat has values BW (includes In-Process, Acquired, or Purchased Research and Development) or BV (includes In-Process, Acquired, or Purchased Research and Development and engineering expense or customer- or government-sponsored research and development). Thus, we remove RDIP from XRD whenever it is included by Compustat, because, as a write-off, it should not be interpreted as an investment. Note also that RDIP is coded by Compustat as negative, so to remove RDIP from XRD we subtract the absolute value of RDIP from XRD.

Brand capital data: The construction of the brand capital stock is analogous to the construction of the knowledge capital stock. Following Belo et al. (2014) and Vitorino (2014), we construct the firm's stock of brand capital from past expenditures data on advertising (Compustat data item XAD) and using the perpetual inventory model as follows:

$$K_{t+1}^B = K_t^B(1-\delta^B)\frac{P_{t+1}^B}{P_t^B} + I_{t+1}^B, \tag{26}$$

where P_t^B is the advertising price index.[13] The initial stock of brand capital is set to:

$$K_0^B = \frac{I_0^B}{g^B + \delta^B - \pi^B(1-\delta^B)},$$

in which I_0^B is the firm's investment in brand capital in the first year in the sample, and π^B is the average (net) growth rate of the price index for advertising expenses, which is 5.4% in the sample period used for the estimation. We let g^B be industry specific and set it to be equal to the average growth rate of advertising expenses in that industry (using the same 10-industry classification described in the construction of the knowledge capital stock). As in Vitorino (2014), we use a depreciation rate for brand capital of 20%. Once we have the initial capital stock, we iterate forward using the depreciation rate, the advertising expenses, and the investment price index. The investment rate on brand capital is then given by the ratio of the current period investment and the beginning of the period corresponding brand capital stock I_t^B/K_t^B.

We note that the total amount of intangible capital (here, knowledge capital and brand capital) of a firm is given by the sum of externally acquired and internally created intangible capital, but our measures of knowledge and brand capital relate to internally created intangible capital only. Consistent with the approach in previous studies, we focus on internally created intangible capital due to several data limitations as discussed in Appendix B.

Additional firm-level variables: We measure debt, B_{it}, as net total debt. Specifically, we measure net debt as long-term debt (Compustat data item DLTT) plus short-term debt (data item DLC), minus cash (data item CHE), setting these items to zero when they are missing. The market value of equity, P_{it}, is the closing price per share (data item PRCC_F) times the number of common shares outstanding (data item CSHO). For firms with different fiscal-year ends the price matches the firm's fiscal year (and thus the timing of the accounting data).

We measure the tax rate, τ_t, as the statutory corporate income tax for the highest bracket from the Commerce Clearing House, annual publications, until 2010, and from Deloitte's corporate tax rates annual publications after 2010. Stock variables with subscript t ($t+1$ for debt) are measured and recorded at the end of year t, while flow variables with subscript t are measured over the course of year t and recorded at the end of year $t+1$.

[13] According to Compustat, advertising is usually an indirect operating cost that is reported by companies as a selling expense within SG&A. Whenever advertising expenditures are reported separately from SG&A (in a note or in a supplementary schedule in the 10-K reports), Compustat includes them in data item XAD. Ptok et al. (2018) discuss in detail the use of several Compustat variables to operationalize various marketing-related constructs. Their results suggest that XAD from Compustat is a satisfactory measure of advertising spending.

4.5. Summary statistics

Panel A in Table 1 reports key summary statistics of the observed valuation ratios and their model-implied components according to Eq. (21), in the pooled sample where all firms are included, and for the low- and high-skill industries separately. All ratios are winsorized at the top and bottom (if the variable admits negative values) 1% to mitigate the impact of outliers in the analysis.

The average valuation ratio across all firms is 1.95. This valuation ratio is higher in high-skill industries than in low-skill industries, 2.04 versus 1.57, respectively.

In terms of the average size of the scaled capital and labor inputs, in the pooled sample, the largest scaled input is labor (using lagged wages as implied by Eq. (21)), which amounts to 61% of total assets. The second largest input is physical capital, with 42% of total assets. The ratio of the knowledge capital stock to total assets is 38%. The smallest capital stock is brand capital with 10% of total assets. The relative magnitude of the ratios varies across the different labor-skill industries.

According to Eqs. (17) to (20), the investment/hiring rates determine the shadow prices of the labor and capital inputs. Panel A in Table 1 shows that, in the pooled sample, investment in knowledge capital has the highest average rate (28%), while investment in labor, the gross hiring rate, has the lowest rate (16%). The investment and hiring rates are all higher and more volatile in high-skill than in low-skill industries. Panel B in Table 1 also reports the investment and hiring rates cross-correlations in the low- and high-skill industries. The table shows that, as expected, the investment/hiring rates are all positively correlated among each other. The correlations range between 14% and 51%. These correlations are significantly smaller than one, thus suggesting that there is at least some independent variation in the shadow prices, and hence the market values, of the different capital and labor inputs in the data.

5. Estimation results

This section reports the main empirical findings.

5.1. Firm value decomposition based on book-values

Before estimating the model, we make a preliminary assessment of the relative importance of each input for firm value based on the book-value of the inputs. We can then use these book-value input-shares as a benchmark to interpret the market-value input-shares obtained from the estimated model.

If adjustment costs are zero, the shadow prices of the capital and labor inputs in Eqs. (17) to (20) are simply one (physical capital), zero (labor), $(1-\tau_t)$ (knowledge capital), and $(1-\tau_t)$ (brand capital). As a result, the value of each capital input is given by its book-value (adjusting for the tax rate), and the fraction of firm value attributed to

Table 1

Descriptive statistics. Notes: Panel A reports the time-series average of the cross-sectional median, and the standard-deviation of selected characteristics of the firm-level data across all firms in the economy, and across the low- and high-skill industries. Panel B reports the cross-correlations of the investment/hiring rates in each industry. VR_{it} is the firm's valuation ratio, I^P_{it}/K^P_{it} is the investment rate in physical capital, H_{it}/L_{it} is the investment rate in labor stock (gross hiring rate), I^K_{it}/K^K_t is the investment rate in knowledge capital, and I^B_{it}/K^B_t is the investment rate in brand capital. We also present the descriptive statistics for the stock variables of each input (physical capital, labor, knowledge capital, and brand capital) relative to the sum of the three capital inputs (A_{it}, total assets as defined in Section 4.2) and relative to annual sales (Y_{it}). The sample consists of firm-level annual data from 1975 to 2016.

Panel A: Averages and standard deviations

	Average			S.D.		
	All firms	Low skill	High skill	All firms	Low skill	High skill
	Valuation ratios					
VR_{it}	1.95	1.57	2.04	4.16	2.83	4.36
	Scaled capital and labor ratios					
K^P_{it}/A_{it}	0.42	0.63	0.38	0.26	0.24	0.25
$(W_{it-1}L_{it})/A_{it}$	0.61	0.54	0.63	1.28	1.62	1.20
K^K_{it}/A_{it}	0.38	0.13	0.44	0.27	0.17	0.26
K^B_{it}/A_{it}	0.10	0.14	0.09	0.16	0.19	0.14
	Investment/hiring rates					
I^P_{it}/K^P_{it}	0.23	0.15	0.26	0.73	0.48	0.77
H_{it}/L_{it}	0.16	0.15	0.17	0.27	0.24	0.28
I^K_{it}/K^K_{it}	0.28	0.21	0.30	0.24	0.18	0.24
I^B_{it}/K^B_{it}	0.25	0.24	0.26	0.24	0.19	0.25
	Capital and labor relative to sales					
K^P_{it}/Y_{it}	0.20	0.25	0.19	0.37	0.31	0.38
$(W_{it-1}L_{it})/Y_{it}$	0.34	0.25	0.36	0.32	0.20	0.34
K^K_{it}/Y_{it}	0.17	0.05	0.21	3.20	0.76	3.48
K^B_{it}/Y_{it}	0.05	0.06	0.05	0.21	0.20	0.21

Panel B: Correlations

	Low skill			High skill		
	H_{it}/L_{it}	I^K_{it}/K^K_{it}	I^B_{it}/K^B_{it}	H_{it}/L_{it}	I^K_{it}/K^K_{it}	I^B_{it}/K^B_{it}
I^P_{it}/K^P_{it}	0.49	0.29	0.35	0.51	0.41	0.36
H_{it}/L_{it}		0.14	0.24		0.29	0.32
I^K_{it}/K^K_{it}			0.35			0.47

each capital input (input-shares) can be computed from Eqs. (12) to (15). In the case of labor, without labor adjustment costs, the shadow price (and hence, its contribution to firm value) is zero because firms cannot sell nor buy workers in the same manner that they buy or sell capital goods.

To characterize the data in a comprehensive yet parsimonious manner, we summarize the properties of the firm-level input-shares in the economy using two different procedures. In the first procedure, we compute an aggregate-level (or industry-level) share (denoted "Aggregate"). Specifically, we separately sum up the numerator and the denominator of Eqs. (12) to (15) across all the firms in the economy (or industry), calculate the share of each input using those aggregated values, and report the corresponding time-series average of the aggregate shares. This aggregate measure provides a straightforward way to interpret the data but puts significantly more weight on the large firms in the economy. Thus, in the second procedure, we compute in each year and for each input, the cross-sectional median input-shares, and report the time series mean of these input-shares (denoted "Average") for each input, properly adjusted to add up to 100%.[14]

Table 2 reports the firm's book-value decomposition using the two procedures for the pooled sample, and separately for the low- and high-skill industries. As it turns out, for the book-value decomposition, the aggregate and the average input-share measures provide a similar characterization of the data, so we focus the discussion here on the aggregate level measure.

As noted, without labor adjustment costs, the value of the installed labor force is zero.[15] In the pooled sample, the most important input is physical capital, which represents about 64.3% of firms' book-value, followed by knowledge capital, which represents 22.8% of firms' book-value, and then brand capital, which represents about 12.9% of

[14] An adjustment is required here because if we compute directly the cross-sectional median share of each input and report the time-series mean of these input-shares, the sum of the shares does not add to 100% because the medians are not additive. Thus, we proceed as follows. First, in each year, we compute the median scaled value of each input (for example, for physical capital, this corresponds to the cross sectional median of $q^P_{it}\frac{K^P_{it+1}}{A_{it+1}}$), then we compute the implied median total firm value as the sum of the median value of each input, and finally we compute the corresponding input-shares as the ratio of the median scaled values of each input as a fraction of the total median firm value. We then report the time-series mean of this measure for each input.

[15] We note, however, that most of the R&D expenses used to construct the knowledge capital stock are labor compensation. Specifically, on average, 45% of R&D expenses correspond to salaries of personnel who are engaged in R&D projects. This suggests that the value of the knowledge capital stock also partially captures the value of labor.

Table 2
Firm-value decomposition based on book values. Notes: This table reports the time-series average of the fraction of firm value (input-shares μ) that is attributed to each input based on its book value. This decomposition is done by setting all the adjustment costs to zero. Shares are computed at the aggregate- and average-level according to the procedure described in Section 5.1. $XS-R^2$ is the cross-sectional R^2, $TS-R^2$ is the time-series R^2, and $m.a.e./\overline{VR}$ is the mean absolute valuation error scaled by the absolute value of the ratio. The results are reported for the sample of all firms, and also for the sub-samples of low-, and high-skill industries. The sample consists of firm-level annual data from 1975 to 2016.

	All firms (1)	Low skill (2)	High skill (3)
		Aggregate (%)	
$\bar{\mu}^P$: Physical	64.33	71.66	61.49
$\bar{\mu}^L$: Labor	0.00	0.00	0.00
$\bar{\mu}^K$: Knowledge	22.78	10.11	27.62
$\bar{\mu}^B$: Brand	12.89	18.23	10.89
		Average (%)	
$\bar{\mu}^P$: Physical	55.97	76.97	51.64
$\bar{\mu}^L$: Labor	0.00	0.00	0.00
$\bar{\mu}^K$: Knowledge	35.69	10.88	40.58
$\bar{\mu}^B$: Brand	8.34	12.15	7.78
		Model fit	
$XS-R^2$	-7.21	-7.63	-7.27
$TS-R^2$	-2.51	-1.78	-2.57
$m.a.e./\overline{VR}$	0.76	0.69	0.77

firms' book-value. These numbers vary significantly across industries. The importance of physical capital and brand capital for the book value of the firm is significantly higher in the low-skill than in the high-skill industries, with 71.7% versus 61.5%, respectively, for physical capital, and with 18.2% versus 10.9%, respectively, for brand capital. Conversely, the importance of knowledge capital for the book value of the firm is significantly lower in low-skill than in high-skill industries, with 10.1% versus 27.6%, respectively. Thus, consistent with the summary statistics of the scaled capital and labor ratios reported in Table 1, Panel A, this decomposition is suggestive that the relative importance of the two intangible capital inputs varies across industries: knowledge capital is more important in high-skill than in low-skill industries, while brand capital is more important in low-skill than in high-skill industries.

The bottom panel in Table 2 provides three measures of model fit. Specifically, the panel reports: i) the cross-sectional R^2 (denoted $XS-R^2$) of the best linear fit of the average portfolio-level valuation ratio plotted against the average portfolio-level predicted valuation ratio; ii) the time-series R^2 (denoted $TS-R^2$) of the pooled portfolio-level data (this measure is the square of the correlation between the predicted and the realized portfolio-level valuation ratios, pooling the data for all portfolios as one large time series); and iii) the average mean absolute error (m.a.e.), computed as the time series mean of the absolute value of the error term of each portfolio (time series average of $|VR_{jt}-\overline{VR_{jt}}|$), as a fraction of the average absolute value of the valuation ratio of the portfolio (denoted m.a.e./$\overline{\text{VR}}$).

According to the three metrics considered here (and, in particular, given the negative cross-sectional and time-series R^2's), the model without adjustment costs in which variation in firm value is only driven by variation in the book value of the inputs is unable to capture the cross-sectional and time-series variation of the valuation ratios of the portfolios. This result suggests that variation in the shadow price of the inputs is likely to be an important ingredient for the ability of the model to capture the large cross-sectional and time-series variation in the valuation ratios observed in the data.

5.2. Parameter estimates and model fit

Table 3, column (1), reports the adjustment costs parameter estimates of the model in the pooled sample. These estimates are $\theta_P=1.50$ for physical capital, $\theta_L=11.26$ for labor, $\theta_K=12.47$ for knowledge capital, and $\theta_B=3.24$ for brand capital. The estimates are all positive, and are statistically significant for labor and knowledge capital, which implies that we cannot reject the hypothesis that these inputs are subject to positive adjustment costs.

The model fit is good, both in the cross-sectional and in the time-series dimensions. Table 3 shows that the cross sectional R^2 is high, 94%, even though the model estimation does not explicitly target this moment. The time-series R^2 is 61%. In terms of average valuation ratio errors, the scaled mean absolute error (m.a.e./$\overline{\text{VR}}$) is quite low, about 22%. Thus, the model explains about 78% of the variation in the portfolio-level observed valuation ratios (the remaining 22% reflect, for example, measurement and misspecification errors). This good fit stands in sharp contrast with the poor fit of the version of the model without adjustment costs reported in Table 2.

Turning to the analysis of the separate estimation of the model in low- and high-skill industries, Table 3, columns (2) and (3) show that all the adjustment cost parameters are positive and statistically significant except for brand capital in the high-skill industries. The estimates of the adjustment costs parameters for labor increase with the average labor-skill level of the industry, with a value of $\theta_L=7.66$ in the low-skill industries compared to $\theta_L=10.64$ in the high-skill industries. Going in the opposite direction, the adjustment costs parameters for physical capital, knowledge capital, and brand capital decrease with the average labor-skill level of the industry.

The model is particularly good at capturing the time-series variation in the valuation ratios in the high-skill industries, with a time-series R^2 of 60%, whereas the time-series fit in the low-skill industries is more modest, with an R^2 of 38%. The cross-sectional fit is quite good in both industries, with a cross-sectional R^2 above 94%. Fig. 1 provides a visual description of the good fit of the model in the cross section. This figure shows the scatter plot across portfolios of the time-series average of the cross-sectional mean valuation ratios observed in the data against the corresponding value predicted by the model. Most portfolios are close to the 45° line. The model mean absolute error in the high-skill industries is low, 22% of the average observed valuation ratio in those industries, and in the low-skill in-

Table 3
Parameter estimates and model fit. Notes: This table reports the parameter estimates and measures of fit for the baseline model specification (columns 1 to 3) and across restricted alternative model specifications with a sub-set of the capital/labor inputs (columns 4 to 13). The estimation uses 40 portfolios sorted based on proxies of the lagged values of the inputs (10 portfolios for each input). θ_P, θ_L, θ_K and θ_B are, respectively, the physical capital, labor, knowledge capital, and brand capital adjustment cost parameters. s.e. stands for Newey-West standard errors with three lags. $XS-R^2$ is the cross-sectional R^2, $TS-R^2$ is the time-series R^2, and $m.a.e./\overline{VR}$ is the mean absolute valuation error scaled by the absolute value of the ratio. The results are reported for the sample of all firms (baseline model only), and also for the sub-samples of low-, and high-skill industries (all specifications). The sample is annual data from 1975 to 2016.

	Baseline			K^P		K^P+L		K^P+K^K		K^P+K^B		K^P+L+K^K	
	All firms (1)	Low skill (2)	High skill (3)	Low skill (4)	High skill (5)	Low skill (6)	High skill (7)	Low skill (8)	High skill (9)	Low skill (10)	High skill (11)	Low skill (12)	High skill (13)
	Parameter estimates												
θ_P	1.50	3.77	2.18	20.55	29.44	10.67	12.79	12.31	13.31	13.59	24.83	5.09	2.68
s.e.	[1.00]	[0.85]	[0.97]	[0.53]	[0.62]	[0.96]	[1.13]	[0.70]	[0.88]	[0.71]	[1.07]	[0.89]	[0.90]
θ_L	11.26	7.66	10.64			10.96	14.54					9.04	10.97
s.e.	[0.69]	[0.84]	[0.66]			[0.97]	[1.02]					[0.84]	[0.67]
θ_K	12.47	18.80	12.28					28.82	14.71			25.14	12.63
s.e.	[0.77]	[1.62]	[0.74]					[2.06]	[0.82]			[1.59]	[0.72]
θ_B	3.24	9.99	2.05							18.92	15.95		
s.e.	[2.05]	[1.46]	[2.48]							[1.78]	[3.14]		
	Model fit												
$XS-R^2$	0.94	0.95	0.94	0.5	0.75	0.57	0.74	0.73	0.88	0.64	0.81	0.86	0.92
$TS-R^2$	0.61	0.38	0.60	-0.01	0.21	0.14	0.39	0.21	0.5	0.20	0.25	0.31	0.6
$m.a.e./\overline{VR}$	0.22	0.31	0.22	0.39	0.32	0.36	0.28	0.34	0.25	0.35	0.32	0.32	0.22

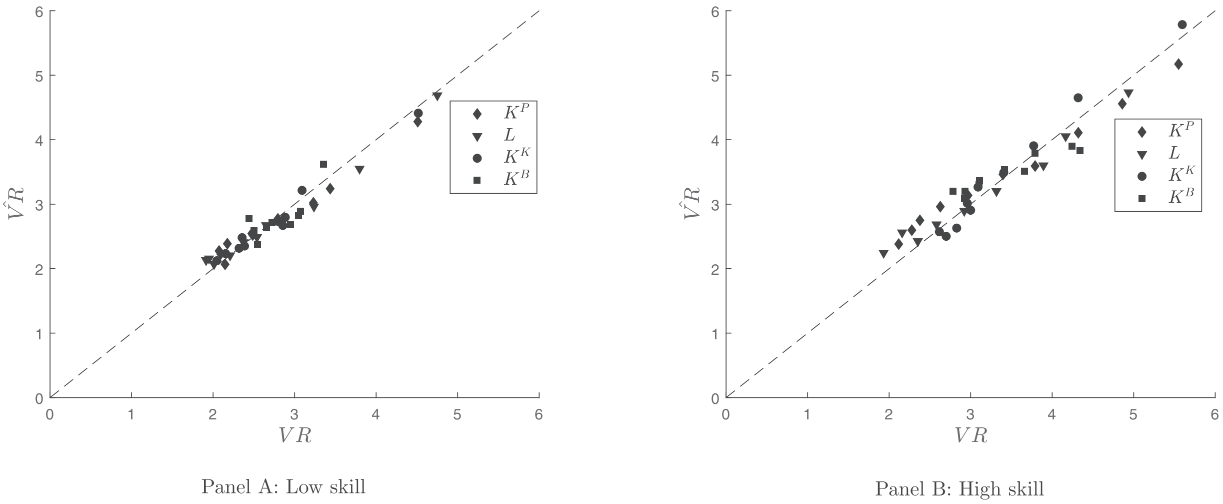


Fig. 1. Cross-sectional model fit. This figure plots the time-series average of the model-implied and realized cross sectional average valuation ratios of each portfolio, using the parameter estimates reported in Table 3 columns (2) (low-skill industries) and (3) (high-skill industries) to calculate the model-implied valuation ratios. The sample consists of firm-level annual data from 1975 to 2016.

dustries it is about 31% of the average observed valuation ratio.

The better time series fit of the model in high-skill industries is consistent with the findings in Peters and Taylor (2017) and Andrei et al. (2018) who show that the observed valuation ratios (market to book ratios) explain physical capital investment better in high-tech sectors than in low-tech sectors.Part of the reason for this pattern can be explained by the significantly higher volatility of the valuation ratios in the high-skill industries (which implies more variation to explain), as reported in Table 1, especially given that this higher volatility does not seem to be driven by random noise but rather by a higher variance in the value of the capital (and labor) inputs. (Panel A in Table 1 shows that the characteristics of the firms in the high-skill industries are more volatile.)

5.3. Firm value decomposition based on market-values

The parameter estimates allow us to compute the model-implied shadow prices of each input, and hence evaluate the contribution of each input for firm value (input-shares) based on each input's market value. The approach used here is analogous to the analysis of the book-value decomposition reported in Section 5.1. Specifically, using the estimates reported in Table 3 columns (1) to (3),

Table 4

Firm-value decomposition and adjustment costs. Notes: This table reports the model-implied input-shares (μ) and estimated adjustment costs (CX/Y) for the baseline model specification (columns 1–3) and across restricted alternative model specifications with a sub-set of the capital/labor inputs (columns 4 to 13), using the parameters estimates reported in Table 3 to calculate the model-implied input-shares and adjustment costs. Shares are computed at the aggregate- and average-level according to the procedure described in Section 5.1. CX/Y is the ratio (in percent) of the implied input adjustment costs-to-sales ratio, computed as the time-series average of the cross-sectional median of this value. The results are reported for the sample of all firms (baseline model only), and also for the sub-samples of low-, and high-skill industries (all specifications). The sample consists of firm-level annual data from 1975 to 2016.

	Baseline			K^P		K^P+L		K^P+K^K		K^P+K^B		K^P+L+K^K	
	All firms (1)	Low skill (2)	High skill (3)	Low skill (4)	High skill (5)	Low skill (6)	High skill (7)	Low skill (8)	High skill (9)	Low skill (10)	High skill (11)	Low skill (12)	High skill (13)
	Firm-value decomposition — Aggregate (%)												
$\bar{\mu}^P$: Physical	30.36	40.16	29.91	100.00	100.00	73.62	67.83	67.72	55.48	61.03	82.86	50.31	32.5
$\bar{\mu}^L$: Labor	22.53	14.33	20.85			26.38	32.17					19.64	22.08
$\bar{\mu}^K$: Knowledge	38.28	20.34	43.23					32.28	44.52			30.05	45.41
$\bar{\mu}^B$: Brand	8.83	25.17	6.02							38.97	17.14		
	Firm-value decomposition — Average (%)												
$\bar{\mu}^P$: Physical	21.85	42.64	20.91	100.00	100.00	68.37	52.94	67.59	44.75	71.52	85.82	48.06	22.19
$\bar{\mu}^L$: Labor	26.61	18.14	24.32			31.63	47.06					22.18	25.43
$\bar{\mu}^K$: Knowledge	46.84	22.19	51.36					32.41	55.25			29.76	52.37
$\bar{\mu}^B$: Brand	4.70	17.03	3.41							28.48	14.18		
	Realized adjustment costs (% of annual sales)												
CP/Y : Physical	0.90	1.22	1.50	6.64	20.23	3.45	8.79	3.98	9.14	4.39	17.06	1.65	1.84
CL/Y : Labor	6.46	2.61	6.77			3.74	9.25					3.09	6.98
CK/Y : Knowledge	10.05	2.35	13.28					3.60	15.91			3.14	13.66
CB/Y : Brand	0.49	1.69	0.30							3.19	2.37		

we compute, for each firm and in each year, the values of $q^P_{it}\frac{K^P_{it+1}}{A_{it+1}}$, $q^L_{it}\frac{L_{it+1}}{A_{it+1}}$, $q^K_{it}\frac{K^K_{it+1}}{A_{it+1}}$, and $q^B_{it}\frac{K^B_{it+1}}{A_{it+1}}$, that is, the model-implied scaled value of each capital/labor input. We then substitute these values in Eqs. (12) to (15) to compute, in each year, the share of the firm's value attributed to each capital/labor input (input-shares), both for the pooled sample, and for the low- and high-skill industries separately.[16]

In Table 4 we summarize the properties of the input-shares in the economy using the two (aggregate and average) measures discussed in Section 2.

Table 4, column (1) shows that the four inputs are important determinants of firms' market values. When the model is estimated across all firms, and using the aggregate input-share measure, the share of physical capital is 30.4%, the share of installed labor is 22.5%, the share of knowledge capital is 38.3%, and the share of brand capital is the remaining 8.8%. When we use the average input-share measure, the inferences are similar, but the importance of physical and brand capital is smaller, while the importance of labor and, especially, of knowledge capital, is higher. Specifically, here, the share of physical capital is 21.9%, the share of installed labor is 26.6%, the share of knowledge capital is 46.8%, and the share of brand capital is the remaining 4.7%. This analysis reveals that physical capital accounts for less than 31% of firms' total market value on average, and highlights the importance of non-physical capital inputs (intangible capital and labor) as determinants of firms' market values.

Turning to the analysis across labor-skill industries, the results reported in Table 4 columns (2) and (3) show that the relative importance of the capital/labor inputs exhibits substantial variation across these industries. The average fraction of firm value attributed to labor and, especially, to knowledge capital, increases with the average labor-skill level of the industry. In the low-skill industries, the share of labor is on average 14.3% using the aggregate measure (18.1% using the average measure), whereas in the high-skill industries this share is 20.9% using the aggregate measure (24.3% using the average measure). Similarly, in the low-skill industries, the share of knowledge capital is on average 20.3% using the aggregate measure (22.2% using the average measure), whereas in the high-skill industries this share is 43.2% using the aggregate measure (51.4% using the average measure).

Going in the opposite direction, the fraction of firm value attributed to physical capital and to brand capital decreases with the average labor-skill level of the industry. In the low-skill industries, the share of physical capital is on average 40.2% using the aggregate measure (42.6% using the average measure), whereas in the high-skill industries this share drops to 29.9% using the aggregate measure (20.9% using the average measure). Similarly, in the low-skill industries, the share of brand capital is on average 25.2% using the aggregate measure (17.0% using the average measure), whereas in the high-skill industries this share drops to 6.0% using the aggregate measure (3.4% using the average measure).

Interestingly, the pattern of the input-shares across industries reveals that, even though intangible capital is an important component of the firm's market value across all

[16] Note that, with this procedure, the input-shares add up to 100% by construction. This does not mean that the model explains the entire variation of the firm's value without any error. As discussed in Section 5.2, based on the m.a.e./$\overline{\text{VR}}$ ratio, the model captures between 69% (low skill) and 78% (high skill) of the firm's valuation ratio. Thus, our analysis here provides a decomposition of the firm value that is explained by the model.

Table 5
Firm-value decomposition across decades. Notes: This table shows the average aggregate input-shares (μ) (obtained with the aggregation procedure described in Subsection 5.1) across different decades, and using the parameter estimates reported in columns (1) to (3) in Table 3. The results are reported for the sample of all firms, and also for the sub-samples of low- and high-skill industries. The sample consists of firm-level annual data from 1975 to 2016.

	1970s	1980s	1990s	2000s	2010s
			All firms (%)		
$\bar{\mu}^P$: Physical	43.15	37.99	28.11	23.93	22.65
$\bar{\mu}^L$: Labor	23.07	18.73	23.01	24.35	24.53
$\bar{\mu}^K$: Knowledge	24.90	33.53	39.89	43.53	44.72
$\bar{\mu}^B$: Brand	8.88	9.75	8.99	8.19	8.10
			Low skill (%)		
$\bar{\mu}^P$: Physical	48.03	47.15	37.41	34.88	35.89
$\bar{\mu}^L$: Labor	14.83	11.17	14.41	16.67	15.22
$\bar{\mu}^K$: Knowledge	17.40	18.80	19.26	23.04	21.97
$\bar{\mu}^B$: Brand	19.74	22.88	28.92	25.42	26.92
			High skill (%)		
$\bar{\mu}^P$: Physical	43.87	37.22	27.57	23.65	21.71
$\bar{\mu}^L$: Labor	21.17	17.68	21.21	22.29	22.73
$\bar{\mu}^K$: Knowledge	28.35	38.03	45.43	48.64	50.34
$\bar{\mu}^B$: Brand	6.60	7.06	5.79	5.43	5.22

industries, the relative importance of the type of intangible capital (knowledge or brand capital) that matters the most for firm value varies across industries. In low-skill industries, the two intangible capital inputs have approximately the same importance (with average input-shares between 17 and 25%) for a combined share of around 39–46% of firm value. But in high skill industries, knowledge capital is significantly more important for firm value than brand capital. Here, the combined share of the two intangible capital inputs is between 49 and 55% of firm value, and knowledge capital accounts for about 90% of this combined share. This result highlights the importance of considering heterogeneous types of intangible capital in empirical work.

5.4. Firm value decomposition over time

The previous analyses focus on the time-series average of the input-shares in the full sample from 1975 to 2016. Using the full-sample parameter estimates, we now perform the decomposition analysis across different subperiods to investigate if the relative importance of the different inputs has changed over time.

Table 5 reports the time series averages of each input-share across decades: 1970s (1975 – 1979), 1980s (1980 – 1989), 1990s (1990 – 1999), 2000s (2000 – 2009), and 2010s (2010 – 2016). To save space, we report only the input-shares computed using the aggregate input-share measure. The results using the average input-share measure are similar, consistent with the analysis in Section 5.3. Fig. 2 provides a visual description of the trends in the input-shares in the data, both in the low- and high-skill industries.

The table and the figure reveal interesting patterns. The importance of physical capital for firm value has significantly decreased in recent years, while the importance of intangible capital, broadly defined, has significantly increased. But the type of intangible capital that has gained more importance in recent years varies across industries. In low-skill industries, there is a significant increase in the importance of brand capital, but a relatively small increase in the importance of knowledge capital. In high-skill industries, there is a significant increase in the importance of knowledge capital, but effectively no change in the importance of brand capital. The importance of labor for firm value does not exhibit an obvious trend as its share has remained relatively constant over the sample period.

Taken together, the analysis in this section further highlights the importance of the non-physical capital inputs for firm value, especially in the most recent decades, and in high-skill industries, where the non-physical capital inputs account, on average, for more than 78% of firm value. In addition, the change in the relative importance of each input for firm value over time confirms the importance of targeting the time series of the valuation ratios in the estimation, as opposed to only targeting their time-series averages as in LWZ/BXZ.

5.5. Implied adjustment costs

To assess whether the model fits the data with economically reasonable parameter values, and also to better understand the relatively high importance of labor and intangible capital inputs for firm value, we use the parameter estimates to characterize the implied adjustment costs of each input.

To properly characterize the adjustment costs of each input we focus on several measures. The first set of measures are based on the realized adjustment costs of each input (that is, ex post adjustment cost measures which describe the equilibrium outcome). Specifically, using the functional form specification in Eq. (16) and the parameter estimates, the realized adjustment costs of each input (denoted as CP, CL, CK, and CB) can be computed as a fraction of firms' total annual sales (denoted as Y) as follows:

$$\frac{CP_{it}}{Y_{it}} = \frac{\frac{\theta_P}{2}\left(\frac{I^P_{it}}{K^P_{it}}\right)^2 K^P_{it}}{Y_{it}} \tag{27}$$

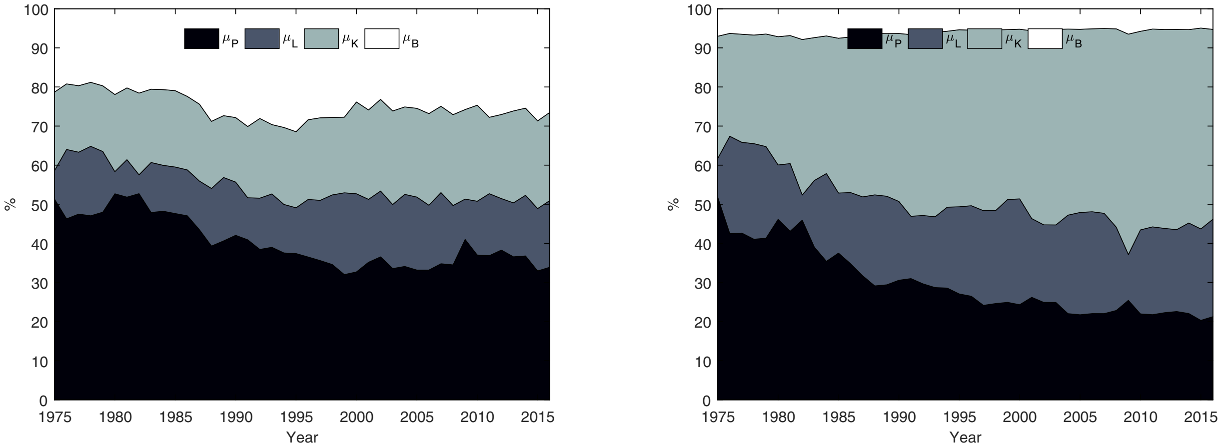
$$\frac{CL_{it}}{Y_{it}} = \frac{\frac{\theta_L}{2}\left(\frac{H_{it}}{N_{it}}\right)^2 W_{it}L_{it}}{Y_{it}} \tag{28}$$

$$\frac{CK_{it}}{Y_{it}} = \frac{\frac{\theta_K}{2}\left(\frac{I^K_{it}}{U^K_{it}}\right)^2 K^K_{it}}{Y_{it}} \tag{29}$$

$$\frac{CB_{it}}{Y_{it}} = \frac{\frac{\theta_B}{2}\left(\frac{I^B_{it}}{U^B_{it}}\right)^2 K^B_{it}}{Y_{it}}. \tag{30}$$

The bottom panel in Table 4, columns (1) to (3), reports the average realized adjustment costs of each input, computed as the time-series average of the cross-sectional medians of the ratios in Eqs. (27)–(30).

Reporting the time-series average of the realized adjustment costs provides a simple way of describing the



Panel A: Low skill

Panel B: High skill

Fig. 2. Firm-value decomposition over time. This figure plots the time series of the contribution of each input for the firm's market value (input-shares) in low-skill industries (Panel A) and in high-skill industries (Panel B) implied by the parameter estimates reported in Table 3, columns (2) and (3), and using the aggregate share measure. μ_P is the share of physical capital, μ_L is the share of labor, μ_K is the share of knowledge capital, and μ_B is the share of brand capital. Panel A shows the results for low-skill industries, Panel B shows the results for high-skill industries. The sample consists of firm-level annual data from 1975 to 2016.

economic importance of the adjustment costs of each input but, given the convexity of the adjustment costs function, the average of the realized values may overstate the actual magnitude of the adjustment costs perceived by firms when making their investment and hiring decisions. Thus, we complement the previous measures with an analysis of the estimated adjustment costs function (that is, an ex ante adjustment costs measure, the relevant object for firms' investment and hiring decisions). Specifically, in Panel A of Fig. 3, we plot the estimated adjustment cost function of each input as a function of the corresponding investment/hiring rate (ranging from −20 to +20%), and holding fixed the median input-to-sales ratio in the industry (to properly scale the function). To further help the economic interpretation of the magnitudes of these functions, Panel B of Fig. 3 reports the estimated adjustment costs of each input using Eqs. (27) to (30) evaluated at their (average) median investment/hiring rates (using the values reported in Table 1).

In the pooled sample, the magnitude of the realized knowledge capital and labor adjustment costs is large, whereas the magnitude of the physical capital and brand capital adjustment costs is very small. The bottom panel in Table 4, column (1), shows that, on average, the fraction of (annual) sales that is lost due to labor adjustment costs is 6.5% while the corresponding fraction for knowledge capital is 10.1%. The fraction of sales that is lost due to physical capital adjustment costs is only 0.9%, and for brand capital this fraction is 0.5%.

Although there is no consensus in the literature regarding the magnitude of adjustment costs, our estimated values of the adjustment costs appear to be reasonable. For labor and physical capital, our estimated values are within the empirical estimates surveyed in Hamermesh and Pfann (1996) and discussed in Merz and Yashiv (2007). For brand capital, the estimated value of adjustment costs is lower than those estimated in Vitorino (2014) (on average, about 8% of firm's annual sales).[17] For knowledge capital, the estimated large adjustment costs are in line with a large body of empirical evidence (for example, Hall et al., 1984 and Bernstein and Nadiri, 1989) documenting that adjustment costs for R&D are significantly higher than those for physical capital.

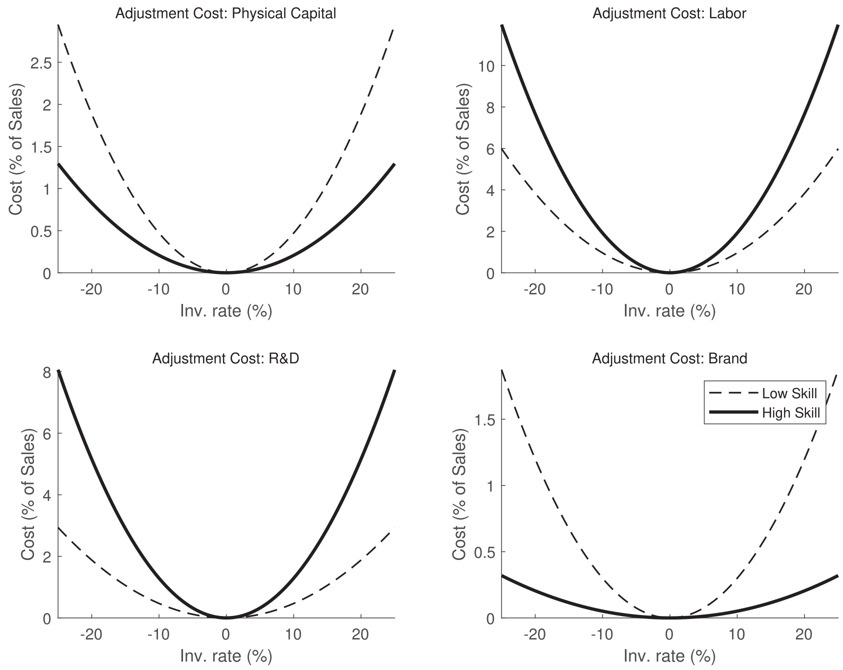
Turning to the analysis of the variation in the size of the adjustment costs across industries, the bottom panel in Table 4, columns (2) to (3), shows that the estimated labor and knowledge capital adjustment costs increase significantly with the average labor-skill level of the industry. The fraction of (annual) sales lost due to labor adjustment costs is on average 2.6% in the low-skill industries, and 6.8% in the high-skill industries. This result is consistent with prior evidence (discussed in Section 4.3) that high-skill workers are more costly to replace than low-skill workers, which motivated our industry classification. Similarly, the fraction of (annual) sales lost due to knowledge capital adjustment costs is on average 2.4% in the low-skill industries, and 13.3% in the high-skill industries. Thus, adjusting knowledge capital is quite costly, but only in high-skill industries.

The positive relationship between the size of the adjustment costs and the average labor-skill of the industry is reversed for brand capital, but the size of the adjustment costs is quite small in both industries (between 0.3 and 1.7% of annual sales), consistent with the analysis across all firms reported in column (1).

Panel A of Fig. 3 plots the estimated adjustment cost functions of each input for the low- and high-skill industries. These plots show that labor and knowledge capital are (ex ante) relatively more costly to adjust in high-skill

[17] The main differences between this study and ours are that we estimate firm-level parameters and a model with four inputs, whereas Vitorino (2014) estimates portfolio-level parameters and a model with only two (physical capital and brand capital) inputs.

Panel A: Adjustment cost functions



Panel B: Adjustment cost functions evaluated at (average) median investment/hiring rates (in %)

	Low skill	High skill
CP/Y : Physical	1.06	1.40
CL/Y : Labor	2.16	5.54
CK/Y : Knowledge	2.07	11.61
CB/Y : Brand	1.73	0.35

Fig. 3. Estimated adjustment cost functions. Panel A in this figure plots the estimated adjustment cost functions for each input in low- and high-skill industries, using the parameter estimates reported in Table 3, columns (2) and (3). The adjustment costs of each input are calculated as a proportion of the respective (average) median input stock-to-sales ratio reported in Table 1. Panel B shows the adjustment costs-to-sales ratio evaluated at the (average) median of corresponding investment rates and as a proportion of the respective (average) median input stock-to-sales ratio reported in Table 1.

industries than in low-skill industries, while the opposite pattern is observed for physical capital and brand capital, consistent with the analysis of the realized adjustment costs. In addition, Panel B shows that the magnitude of the (ex ante) adjustment costs appears to be reasonable and is similar to that using the realized (ex post) measures in our sample.

Taken together, the adjustment costs estimates reveal that knowledge capital and labor are the most costly inputs to adjust in response to changing economic conditions. This finding helps understand the high share of these inputs in the model-implied firm-value decomposition, especially in high-skill industries.

5.6. Model comparison

To help understand the fit of the model and the relative importance of the various capital inputs for firm valuation, Table 3, columns (4) to (13), reports the parameter estimates and model fit across several restricted versions of the model based on various subsets of the four inputs. Table 4, columns (4) to (13), reports the corresponding model-implied firm value decomposition and adjustment cost estimates. To save space, for each of the alternative specifications, we only report the results for the low- and high-skill industries. Also, we focus most of the discussion on the comparison of the model fit across specifications. To provide a meaningful comparison of the model fit in terms of R^2, we use the same set of firms in the estimation of all models (the sample used for the baseline model), and the same observed valuation ratios across models, that is, the observed firm value scaled by the same sum of the capital inputs (A_t).

The standard one-physical-capital input model is a natural benchmark. Table 3, columns (4) and (5), show that, consistent with BXZ, this model does a reasonable job explaining the cross-sectional variation in the average valuation ratio across portfolios with a cross-sectional R^2 of 50% in low-skill industries, and of 75% in high-skill industries (versus 95 and 94% in the baseline model). However, the one-physical-capital input model fails to explain the time-series variation in the valuation ratios. The time-series R^2 of the one-physical-capital input model is basically 0% in low-skill industries, versus 38% in the baseline model, and 21% in high-skill industries, versus 60% in the baseline model. Thus, we conclude that the benefit of incorporating additional quasi-fixed inputs in the neoclassical investment model comes primarily from improving the model's ability to capture the time-series variation in the valuation ratios. In addition, this result highlights the importance of examining the time series fit of the model to assess its performance, not just its cross sectional fit as in LWZ/BXZ.

In addition, the comparison of the estimated realized adjustment costs across model specifications reveals that, when some inputs are ignored, the model estimates provide an improper characterization of the size of an input's adjustment cost. As reported in the bottom panel in Table 3, the one-physical-capital input model seems to significantly overestimate the magnitude of physical capital adjustment costs, suggesting that it attributes to physical capital all the costs of adjusting the other (missing) inputs. In this model, the fraction of annual sales lost due to physical capital adjustment costs is on average between 6.6 and 20.2% of firms' annual sales across industries, versus only 1.2 and 1.5% of firms' annual sales in the baseline model.

Comparing across all model specifications, Table 3 shows that the contribution of each input for the improvement of the model fit varies across industries. We focus our discussion here on the time-series R^2 only, because this metric is the most informative for this analysis due to its larger variation across model specifications.

Adding labor and, especially, knowledge capital, to the one-physical-capital input model has a significant impact on the quality of the model fit in both industries, whereas adding brand capital has a significant impact on the quality of the model fit in low-skill industries only. When quasi-fixed labor is added to the one-physical-capital input model, the time-series R^2 in low-skill industries increases from 0 to 14% (columns 4 and 6), and it increases from 21 to 39% (columns 5 and 7) in high-skill industries. The impact of knowledge capital is even stronger. The time-series R^2 in low-skill industries increases from 0 to 21% (columns 4 and 8), and increases from 21 to 50% (columns 5 and 9) in high-skill industries, when knowledge capital is added to the one-physical-capital input model. Finally, when brand capital is added to the one-physical-capital input model, the time-series R^2 in low-skill industries increases significantly from 0 to 20% (columns 4 and 10), while in high-skill industries it increases very little from 21 to 25% (columns 5 and 11).

5.7. Robustness

To check the robustness of our main findings and, in particular, the high importance of non-physical capital inputs for firm value, we re-estimate the model using different functional forms, estimation approaches, data sample, definitions of intangible capital, and other variations of the empirical procedures (in addition to the robustness checks already discussed in the previous sections). To save space, we report the full set of results in the online appendix, and we briefly summarize the main conclusions from these analyses here.

First, we estimate the model assuming more general adjustment costs functions. In one specification, we allow adjustment costs to be asymmetric to capture, for example, investment irreversibility as in Abel and Eberly (1994) and Abel and Eberly (1996). In another specification, we also estimate the curvature parameter in the adjustment costs function as in BXZ, thus allowing the function to depart from the quadratic form. We find that allowing for asymmetry or a flexible curvature in the adjustment costs function has a small impact on the quality of the model fit in our sample and hence on the conclusions derived from the estimation of the baseline model.

Second, we estimate the model using different estimation approaches. We consider a larger number of portfolios than in the baseline estimation (80 versus 40 portfolios, respectively) and we also estimate the model directly using firm-level data (as opposed to performing the estimation using portfolios). In addition, we also estimate the model using the alternative Fama and French 17 industry classification. Finally, we estimate the model parameters using the Euler equation approach at the firm-level. We find that, similar to the baseline estimation, the non-physical capital inputs still account for more than 60% of the firm's market value across these different estimation approaches.

Third, we consider a different sample for the estimation. We re-estimate a restricted version of the model without knowledge capital using the sub-sample of firms that were excluded from the main sample due to missing (or always zero) R&D expenses data, but have physical capital, labor, and brand capital data. This alternative sample is quite large, including 6541 firms and 60,316 firm-year observations. We find that the average contribution of the non-physical capital inputs for firm value in this alternative sample is still more than 38% of firms' market value across industries. This lower share relative to the baseline sample is perhaps not surprising given that, by definition, the non-R&D firms have zero knowledge capital, which (across most specifications) is the non-physical capital input that contributes the most for firm value in the baseline sample. In addition, the firms that do not perform R&D are likely to be firms from the "old economy," and naturally rely less on innovation and other intangibles.

Finally, we consider different definitions of intangible capital in the empirical implementation of the model. We estimate the model using an alternative (broader) measure of intangible capital, known as organization capital, which is constructed based on Selling, General and Administrative (SG&A) expenses data, following the approach in Eisfeldt and Papanikolaou (2013). We find that, although this SG&A measure of intangible capital does not allow us to differentiate across types of intangibles, as discussed previously, the estimated decomposition across physical capital, labor and intangible capital is quite similar to that of the baseline model. In addition, when we estimate an augmented version of the model adding organization capital as a fifth input, we find that the overall fit of this augmented model is similar to the fit of the more parsimonious baseline model.

Overall, these additional robustness checks confirm that the importance of the non-physical capital inputs for firm value is a finding robust to reasonable variations in the empirical procedures.[18]

6. Conclusion

We incorporate quasi-fixed labor, knowledge capital, and brand capital into the neoclassical model of investment, and estimate the contribution of each input for explaining firm market values in the US economy from 1975 to 2016. The model performs well in explaining both cross-sectional and time-series variation in firms' market values across industries, with a time-series R^2 of up to 61%, and a cross-sectional R^2 of up to 95%. We find that the importance of the non-physical inputs for firm value is substantial and varies across industries. On average, while physical capital accounts for 30 to 40% of firms' market value across industries, installed labor force accounts for 14 to 21%, knowledge capital accounts for 20 to 43%, and brand capital for 6 to 25%.

We show that financial markets assign large and positive values to the installed stocks of the different types of inputs because they are costly to adjust, especially labor and knowledge capital, thus allowing firms to extract some rents as compensation for the cost of adjusting the inputs. Overall, our analysis documents the importance of the non-physical capital inputs (labor and intangible capital) for firm value, and provides direct empirical evidence supporting models with multiple inputs as main sources of firm value.

We also document that the contribution of each input for firm value varies over time. The importance of physical capital has decreased substantially over the last four decades, while the importance of knowledge capital input has increased significantly. This trend is pervasive across different industries and thus is not driven by changes in the industry composition in the US economy, but rather reveals a pattern in the overall economy.

Finally, our quantitative analysis uncovers new questions for future research. For example, what makes brand capital relatively more important in low-skill industries than in high-skill industries, whereas the oppositive pattern holds true for knowledge capital? What is the economic source of the large magnitude of adjustment costs in knowledge capital? The answer to these and other questions may help us understand better the valuation of companies in financial markets.

Appendix A. Derivation of the firm value decomposition

The first order conditions with respect to I_{it}^P, K_{it+1}^P, H_{it}, L_{it+1}, I_{it}^K, K_{it+1}^K, I_{it}^B, K_{it+1}^B, and B_{it+1}, from maximizing the cum-dividend market value of equity are:

$$q_{it}^P = 1 + (1-\tau_t)\frac{\partial C_{it}}{\partial I_{it}^P} \tag{A.1}$$

$$q_{it}^P = E_t\left[M_{t+1}\left[(1-\tau_{t+1})\left(\frac{\partial \Pi_{it+1}}{\partial K_{it+1}^P} - \frac{\partial C_{it+1}}{\partial K_{it+1}^P}\right) + \delta_{it+1}^P \tau_{t+1} + (1-\delta_{it+1}^P) q_{it+1}^P\right]\right] \tag{A.2}$$

$$q_{it}^L = (1-\tau_t)\frac{\partial C_{it}}{\partial H_{it}} \tag{A.3}$$

$$q_{it}^L = E_t\left[M_{t+1}\left[(1-\tau_{t+1})\left(\frac{\partial \Pi_{it+1}}{\partial L_{it+1}} - \frac{\partial C_{it+1}}{\partial L_{it+1}} - W_{it+1}\right) + (1-\delta_{it+1}^L) q_{it+1}^L\right]\right] \tag{A.4}$$

$$q_{it}^K = (1-\tau_t)\left[1 + \frac{\partial C_{it}}{\partial I_{it}^K}\right] \tag{A.5}$$

$$q_{it}^K = E_t\left[M_{t+1}\left[(1-\tau_{t+1})\left(\frac{\partial \Pi_{it+1}}{\partial K_{it+1}^K} - \frac{\partial C_{it+1}}{\partial K_{it+1}^K}\right) + (1-\delta_{it+1}^K) q_{it+1}^K\right]\right] \tag{A.6}$$

$$q_{it}^B = (1-\tau_t)\left[1 + \frac{\partial C_{it}}{\partial I_{it}^B}\right] \tag{A.7}$$

$$q_{it}^B = E_t\left[M_{t+1}\left[(1-\tau_{t+1})\left(\frac{\partial \Pi_{it+1}}{\partial K_{it+1}^B} - \frac{\partial C_{it+1}}{\partial K_{it+1}^B}\right) + (1-\delta_{it+1}^B) q_{it+1}^B\right]\right] \tag{A.8}$$

$$1 = E_t\left[M_{t+1}\left[r_{it+1}^B - (r_{it+1}^B - 1)\tau_{t+1}\right]\right] = E_t\left[M_{t+1} r_{it+1}^{Ba}\right]. \tag{A.9}$$

In the last equation we define the after-tax bond return as $r_{it+1}^{Ba} \equiv r_{it+1}^B - (r_{it+1}^B - 1)\tau_{t+1}$.

Using the FOCs (A.2), (A.4), (A.6), and (A.8) we can write:

$$q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K K_{it+1}^K + q_{it}^B K_{it+1}^B$$
$$= E_t\left[M_{t+1}\left[(1-\tau_{t+1})\left(\frac{\partial \Pi_{it+1}}{\partial K_{it+1}^P} K_{it+1}^P + \frac{\partial \Pi_{it+1}}{\partial L_{it+1}} L_{it+1}\right.\right.\right.$$

[18] The results reported in the online appendix also show that the model-implied firm value decomposition is more stable across different empirical specifications of the model than the adjustment-cost parameter estimates. Two features of the analysis contribute to the stability of the firm value decomposition. First, the input shares have to add up to one. This "adding up constraint" (the rescaling of each input value by the total) tends to dampen the changes in input shares induced by changes in adjustment cost parameters. Second, the relative book-value of the input (which is independent of the adjustment cost estimates) is a key anchor (determinant) of the input shares. Each adjustment cost parameter will change the corresponding input's share relative to this anchor through its impact on the *marginal* adjustment cost estimate. However, on average, the marginal adjustment cost is smaller than the unit price of capital (normalized to 1), so it has only a moderate direct effect on the total magnitude of each input's share. The share of labor is an exception because its shadow value depends entirely on the marginal adjustment cost.

$$
\begin{aligned}
&+ \frac{\partial \Pi_{it+1}}{\partial K^K_{it+1}} K^K_{it+1} + \frac{\partial \Pi_{it+1}}{\partial K^B_{it+1}} K^B_{it+1} \Bigg) \\
&- (1-\tau_{t+1}) \Bigg(\frac{\partial C_{it+1}}{\partial K^P_{it+1}} K^P_{it+1} + \frac{\partial C_{it+1}}{\partial L_{it+1}} L_{it+1} + \frac{\partial C_{it+1}}{\partial K^K_{it+1}} K^K_{it+1} \\
&+ \frac{\partial C_{it+1}}{\partial K^B_{it+1}} K^B_{it+1} \Bigg) \\
&+ (1-\delta^P_{it+1}) q^P_{it+1} K^P_{it+1} + (1-\delta^L_{it+1}) q^L_{it+1} L_{it+1} \\
&+ (1-\delta^K_{it+1}) q^K_{it+1} K^K_{it+1} + (1-\delta^B_{it+1}) q^B_{it+1} K^B_{it+1} \\
&+ \delta^P_{it+1} \tau_{t+1} K^P_{it+1} - (1-\tau_{t+1}) W_{it+1} L_{it+1} \Bigg]\Bigg].
\end{aligned}
$$

Given the homogeneity of degree one of the operating profit function and the adjustment costs function, we have:

$$
\begin{aligned}
& q^P_{it} K^P_{it+1} + q^L_{it} L_{it+1} + q^K_{it} K^K_{it+1} + q^B_{it} K^B_{it+1} \\
&= E_t\Big[M_{t+1} \Big[(1-\tau_{t+1}) (\Pi_{it+1} - C_{it+1} - I^K_{it+1} - I^B_{it+1} \\
&\quad - W_{it+1} N_{it+1}) - I^P_{it+1} + \delta^P_{it+1} \tau_{t+1} K^P_{it+1} \\
&\quad + (1-\tau_{t+1}) \frac{\partial C_{it+1}}{\partial I^P_{it+1}} I^P_{it+1} + I^P_{it+1} + (1-\tau_{t+1}) \frac{\partial C_{it+1}}{\partial H_{it+1}} H_{it+1} \\
&\quad + (1-\tau_{t+1}) \frac{\partial C_{it+1}}{\partial I^K_{it+1}} I^K_{it+1} \\
&\quad + I^K_{it+1} + (1-\tau_{t+1}) \frac{\partial C_{it+1}}{\partial I^B_{it+1}} I^B_{it+1} + I^B_{it+1} \\
&\quad + (1-\delta^P_{it+1}) q^P_{it+1} K^P_{it+1} + (1-\delta^L_{it+1}) q^L_{it+1} L_{it+1} \\
&\quad + (1-\delta^K_{it+1}) q^K_{it+1} K^K_{it+1} \\
&\quad + (1-\delta^B_{it+1}) q^B_{it+1} K^B_{it+1} \ \Big]\Big] \\
&= E_t\Big[M_{t+1} \Big[(1-\tau_{t+1}) (\Pi_{it+1} - C_{it+1} - I^K_{it+1} - I^B_{it+1} - W_{it+1} N_{it+1}) \\
&\quad - I^P_{it+1} + \delta^P_{it+1} \tau_{t+1} K^P_{it+1} + B_{it+2} - r^B_{it+1} B_{it+1} \\
&\quad + q^P_{it+1} K^P_{it+2} + q^L_{it+1} L_{it+2} + q^K_{it+1} K^K_{it+2} + q^B_{it+1} K^B_{it+2} - B_{it+2} \ \Big]\Big] \\
&\quad + E_t\Big[M_{t+1} r^B_{it+1} \Big] B_{it+1}.
\end{aligned}
$$

Rearranging the above equation,

$$
\begin{aligned}
& q^P_{it} K^P_{it+1} + q^L_{it} L_{it+1} + q^K_{it} K^K_{it+1} + q^B_{it} K^B_{it+1} - B_{it+1} \\
&= E_t\left[M_{t+1} \left[\begin{array}{c} D_{it+1} + q^P_{it+1} K^P_{it+2} \\ + q^L_{it+1} L_{it+2} + q^K_{it+1} K^K_{it+2} + q^B_{it+1} K^B_{it+2} - B_{it+2} \end{array} \right]\right].
\end{aligned}
$$

Recursively applying the above the equation to future periods,

$$
\begin{aligned}
& q^P_{it} K^P_{it+1} + q^L_{it} L_{it+1} + q^K_{it} K^K_{it+1} + q^B_{it} K^B_{it+1} - B_{it+1} \\
&= E_t\Big[M_{t+1} D_{it+1} + M_{t+2} D_{it+2} + M_{t+2} \Big[q^P_{it+2} K^P_{it+3} + q^L_{it+2} L_{it+3} \\
&\quad + q^K_{it+2} K^K_{it+3} + q^B_{it+2} K^B_{it+3} - B_{it+3} \Big]\Big] \\
&= \ldots \\
&= \sum_{\triangle t=1}^{\infty} M_{t+\triangle t} D_{it+\triangle t} + \lim_{\triangle t \to \infty} E_t\Big[M_{t+1} \Big[q^P_{it+\triangle t} K^P_{it+\triangle t} + q^L_{it+\triangle t} L_{it+\triangle t} \\
&\quad + q^K_{it+\triangle t} K^K_{it+\triangle t} + q^B_{it+\triangle t} K^B_{it+\triangle t} - B_{it+\triangle t} \Big]\Big].
\end{aligned}
$$

Assuming that the transversality condition holds then,

$$
\begin{aligned}
& q^P_{it} K^P_{it+1} + q^L_{it} L_{it+1} + q^K_{it} K^K_{it+1} + q^B_{it} K^B_{it+1} = V_{it} - D_{it} + B_{it+1} \\
&= P_{it} + B_{it+1}.
\end{aligned}
$$

Appendix B. Additional details about the data

Here we present the investment price index data used to construct the capital stocks, discuss the calculation of the industry-level wage rate and quit rate, the treatment of the missing R&D and advertising data, and the labor-skill industry classification. In addition, we discuss some limitations of the intangible capital data.

Investment price index data: There is no readily available data on a price of investment index P^P_t for our physical capital stock, which is mostly composed of structures and equipment. The BEA provides a price index for a broad investment series that includes investment in structures, equipment, and intellectual property products (which should be excluded from our analysis because it corresponds to intangible capital), and for each of these three items separately.[19] Thus, we first recover the real values for each of the series "structures" and "equipment" by dividing the nominal values of these two series reported by the BEA in the National Income and Product Accounts (NIPA) table 5.3.5 by their corresponding price indices reported by the BEA in the NIPA table 5.3.4. We then calculate a price index for physical capital that includes only structures and equipment (but not intellectual property) in the same manner as the BEA constructs price deflators by dividing the nominal-dollar value of a series by its calculated real value. More specifically, we proceed by dividing the sum of the nominal investment in structures and equipment (reported in the NIPA table 5.3.5) by the sum of the real investment in structures and equipment (recovered by us as described above).

The price of investment index P^K_t for R&D capital is the BEA price index for intellectual property products, from the Federal Reserve Economic Data (FRED) database.[20] The price of investment index P^B_t for brand capital is the advertising industry's output price index (PPI), available from the Bureau of Labor Statistics.[21]

[19] The price index for the broad investment series is called "Gross Private Domestic Investment: Fixed Investment: Nonresidential (implicit price deflator)" (series id A008RD3Q086SBEA in FRED).

[20] Specifically, we use the annual series "Gross Private Domestic Investment: Fixed Investment: Nonresidential: Intellectual Property Products: Research and Development (chain-type price index), Index 2009=100" (Y006RG3A086NBEA) provided by the BEA.

[21] Specifically, the price index for brand capital is the average yearly Producer Price Index by Industry: Advertising Agencies (US Bureau of Labor Statistics, "Producer Price Index by Industry: Advertising Agencies" [PCU541810541810], retrieved from FRED, Federal Reserve Bank of St. Louis). Because this data series only starts in 1996, we extrapolate backward using as predictors "Personal Consumption Expenditures: Chain-type Price Index, Index 2009=100 (BEA)," "Gross Private Domestic Investment: Fixed Investment: Nonresidential: Intellectual Property Products (chain-type price index), Index 2009=100," "Gross Private Domestic Investment: Fixed Investment: Nonresidential: Intellectual Property Products: Research and Development (chain-type price index), Index 2009=100 (BEA)," "Gross Private Domestic Investment: Fixed Investment: Nonresidential: Intellectual Property Products: Entertainment, Literary, and Artistic Originals (chain-type price index), Index 2009=100 (BEA)," and "Private fixed investment, chained price index: Nonresidential: Intellectual property products: Software, Index 2009=100 (BEA)" from the period of 1929 until 1995). (Note: the IPP software series only starts in 1959 so it only enters as a predictor after 1959.)

Wage and employee quit-rate data: To compute the wage rate per worker, W_{it}, we use annual data from the BEA-NIPA, Section 6. The industry-level wage rate per worker is given by the ratio of the total compensation of employees (which includes wage and salary accruals and supplements to wages and salaries) to the total number of employees in the industry. We use compensation of employees by industry from Tables 6.2B-D and the number of (full-time and part-time) employees by industry from Tables 6.4B-D. To merge the wage data with our firm-level data from Compustat/CRSP, we created a mapping between the wage data and the Standard Industry Classification (SIC) 1987 and the North American Industry Classification System (NAICS) 2002 codes using the industries' description in the BEA tables.

We measure the annual employee quit rate δ^L_{it} using data for 16 major industry groups based on NAICS codes from the Job Openings and Labor Turnover Survey (JOLTS) available from the Bureau of Labor Statistics (BLS). Because this data is only available since 2001, we extend the data backward as follows. We estimate a time-varying quit rate by regressing, for each major industry group in JOLTS, the industry level quit rates on real GDP growth, unemployment rate, the labor vacancy rate, and a measure of labor-market tightness.[22] The fit of the regression for each industry is quite good, with a median time-series R^2 of 88% across industries. For each industry, we then extend the quit rate back to cover the entire sample prior to 2001. We also use the same procedure to estimate a time-varying aggregate JOLTS quit rate for the industry group "Total Private" (i.e., an overall quit rate), and assign this rate to firms that belong to industries not covered in JOLTS or that have a missing industry code. This procedure allows us to have variation in the employee quit rate both in the cross-section and in the time-series.

Missing R&D and advertising data: We treat missing R&D data as follows. In 1972, the APB Opinion No. 22 made the disclosure of R&D expenditures in financial statements mandatory and the SEC started to require the reporting of R&D in the Annual 10-K reports (SEC No.125). However, until 1975, when the FASB started to require the expensing of all R&D expenditures during the year incurred, disclosure of R&D in the firms' financial statements was still limited. Given this, after 1975, we assume that, if R&D is missing, it corresponds to zero. Also, to use as much data as possible, and because there is a significant share of companies that reports R&D prior to 1975, we construct the stocks of knowledge capital starting in 1970 (whenever possible). Even though we only estimate the model starting in 1975, this procedure allows us to mitigate the negative impact on our analysis of the potential mis-measurement of the initial knowledge capital stock for the firms present in the sample which have data prior to 1975.

We treat the missing Advertising (item XAD in Compustat) data as follows. In 1994, the SEC passed Financial Reporting Release 44 (FRR 44), which eliminated the disclosure requirement of advertising expenditures in public firms' annual reports. Before the passage of FAR 44 (which became effective on December 20, 1994), public firms were required to report advertising spending if it exceeded 1 percent of their total sales (according to the SEC Release AS-125, which became effective for financial statements for periods ending on or after Dec 31, 1972). Based on this we calculate the brand capital stocks using data starting in 1972 when companies start to report advertising expenses in the "Supplementary Income Statement Information" schedule. We impute missing advertising data based on the observed selling, general and administrative (SG&A) expenses using the firm-level average ratio of advertising expenses-to-SG&A ratio for the years in which neither of these values is missing. Given the different disclosure requirements throughout the years, however, we cap the imputed amount at 1% of sales for the years from 1972 through 1993 (to make the imputed values consistent with the reporting standards). We exclude from the sample all the firms with missing XAD during the entire sample period.

Labor skill industry classification data: We classify an industry to be a low- or high-skill industry based on the percentage of workers in that industry that work on occupations that require a high level of training and preparation (high-skill workers) using the Specific Vocational Preparation (SVP) index from the Dictionary of Occupational Titles (DOT), available from the Department of Labor, and employee data from the BLS, Occupational Employment Statistics (OES) program. The data is from Belo et al. (2017), available from the authors' webpages. The base industry-level data is available at the three-digit SIC level before and including year 2001, and at the four-digit NAICS level after 2001. An industry is classified as a high labor-skill industry if it belongs to a 3-SIC or 4-NAICS industry in which the percentage of high-skill workers in that industry (defined in Belo et al. 2017 as variable PSKILL) is above the median of the cross-sectional distribution of the PSKILL variable. Conversely, we classify an industry as a low labor-skill industry if the percentage of high-skill workers in that industry is below the median of the cross-sectional distribution of the industry-level PSKILL variable. Because the data only refers to the period from 1991 to 2013, for the period from 1975 to 1990, we use an average of the data from 1991 to 2001 and, for the period from 2014 to 2016, we use an average of the data from 2002 to 2013. The industry classification of each firm is very stable over time.

Discussion on the limitations of the intangible capital data: Our measurement of the firm-level intangible capital stocks follows previous work whenever possible taking into account legal reporting requirements and how the data is treated in Compustat. In constructing these stocks, however, it is important to recognize that there are several implicit empirical choices that one has to make, and

[22] For the real GDP growth we use the series: Real Gross Domestic Product (US Bureau of Economic Analysis, series A191RL1A225NBEA, retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/A191RL1A225NBEA, April 30, 2018). As for the unemployment rate we use the series: Civilian Unemployment Rate [UNRATE] (US Bureau of Labor Statistics, retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/UNRATE, April 30, 2018.). For the labor vacancy rate we use the Help Wanted Index (HWI) referenced in Barnichon (2010) and provided in Regis Barnichon's website. The HWI is provided at the monthly level, in the regressions we use the yearly average. We construct a measure of labor market tightness as the ratio of the vacancy rate and the unemployment rate.

which can potentially have some impact on the results. For example, although the total amount of intangible capital (here, knowledge capital and brand capital) of a firm is given by the sum of externally acquired and internally created intangible capital, our measures of knowledge and brand capital relate to internally created intangible capital only. We focus on internally created intangible capital due to several data limitations. First, when one company purchases another, externally generated or acquired intangible assets are recorded by companies as assets in the balance sheet and are recorded by Compustat in item INTAN (Intangibles) which corresponds to the sum of the items INTANO (Other Intangibles) and GDWL (Goodwill). Other intangibles refers to identifiable intangible assets.[23] Goodwill is recorded as the residual between the cost of an acquired business and the fair market value of net tangible assets and identifiable intangible assets. This means that, even though Goodwill is recorded as an intangible asset on the acquiring company's balance sheet, goodwill is often contaminated by non-intangibles (such as market premium for physical assets).

In addition, the treatment and reporting of externally created intangible assets has not been consistent over time. For example, up to 2001, when a firm acquired another company, it could choose to recognize all the intangible assets that were acquired at that time ("purchase method") or only those that had been previously recorded by the acquired entity at the time of a previous acquisition ("pooling-of-interests method"). After 2001, however, with accounting rule SFAS 141, the "pooling-of-interests method" was eliminated as an option. This leads to inconsistency in the treatment of intangibles across and within companies. Moreover, the breakdown of INTAN into "Other Intangibles" and "Goodwill" is only available in Compustat after 2001 (Accounting rule SFAS 142 issued in 2001 requires greater disclosure of information about goodwill and intangible assets).

[23] An asset is identifiable when it is separable, that is, it is capable of being separated or divided from the acquired entity and sold, transferred, licensed, rented or exchanged (regardless of whether there is an intent to do so) or when it arises from contractual or other legal rights. Examples of identifiable intangible assets include computer software, licenses, trademarks, patents, films, copyrights and import quotas.

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