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Brand capital and firm value ☆

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ABSTRACT

We study the role of brand capital – a primary form of intangible capital – for firm valuation and risk in the cross section of publicly traded firms. Using an empirical measure of brand capital stock constructed from advertising expenditures accounting data, we show that: (i) firms with low brand capital investment rates have higher average stock returns than firms with high brand capital investment rates, a difference of 5.2% per annum; (ii) more brand capital intensive firms have higher average stock returns than less brand capital intensive firms, a difference of 5.1% per annum; and (iii) investment in both brand capital and physical capital is volatile and procyclical. A neoclassical investment-based model in which brand capital is a factor of production subject to adjustment costs matches the data well. The model also provides a novel explanation for the empirical links between advertising expenditures and stock returns around seasoned equity offerings (SEO) documented in previous studies.

1. Introduction

The value of a corporation reflects the value of its physical capital stock (e.g. machines, plants), as well as the value of its intangible capital stock (e.g. employee skills, brand name, customer base). As documented in Hall (2001) and McGrattan and Prescott (2000), intangible capital is an important component of aggregate stock market value, and this importance has significantly increased in the last decades. Thus, understanding the effect of intangible capital on firms' performance is an important question which can help us understand the economic determinants of firms' market values. In this paper, we study a specific form of intangible capital, brand capital, and evaluate its effect on firm's risk (expected stock returns) in the cross section of publicly traded firms, both empirically as well as through the lens of a neoclassical investment-based asset pricing model.

Brand capital is an intangible asset that summarizes consumers' awareness of the goods and services produced by a firm. In the spirit of Griliches (1979), we interpret brand capital stock as a factor of production in the firm's operating profit function because it helps firms increase sales through, for example, increased customer loyalty or visibility. In addition,

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brand capital allows firms to differentiate their goods and services from those of competitors and thus it is a potential source of competitive advantage. Through its effect on cash flows, the brand capital stock is likely to affect the risk properties of the firm, and hence its cost of capital and market value.

Being an intangible asset, a firm's brand capital stock is naturally difficult to measure. To construct this variable, we interpret firms' advertising expenditures as investment in brand capital. These expenditures, which at the aggregate level represent about 5% of annual GDP in the U.S. economy (Arkolakis, 2010), include the cost of advertising media and promotional expenses and thus are a natural form through which firms affect brand awareness.[1] Following the literature on intangible capital, we then construct a firm-level measure of brand capital stock from advertising expenditures accounting data using the perpetual inventory method. We use this measure to investigate the link between brand capital and stock returns in the data.

Our main empirical findings can be summarized as follows. First, firms with low brand capital investment rates (low advertising expenditures) have higher average stock returns than firms with high brand capital investment rates (difference of 5.2% per annum). We interpret the difference in average stock returns between firms with different brand capital investment rates as reflecting a compensation for differences in macroeconomic risk of these firms. Supporting this hypothesis, we show that a conditional version of the capital asset pricing model explains a large fraction of the cross-sectional variation in the average returns of firms with different brand capital investment rates. In addition, we show that the cash flows of firms with low brand capital investment rates are relatively more cyclical.

Second, we document that more brand capital intensive firms, i.e. firms with higher stock of brand capital per employee, have higher average returns than less brand capital intensive firms (difference of 5.1% per annum). Finally, we show that advertising expenditures and physical capital investment have similar properties, consistent with interpreting advertising expenditures as an investment. Both series are volatile (especially brand capital investment) and procyclical.

To interpret the empirical findings, we incorporate brand capital into an otherwise standard neoclassical model of investment. We then use the model as a laboratory to understand the economic mechanism driving the empirical facts documented here, as well as to interpret the link between advertising expenditures and seasoned equity offerings (SEOs) documented in previous studies (and discussed below).

The model features a large cross section of firms. Each firm produces a differentiated good and is subject to a firm-specific and an aggregate productivity shock. Firm managers make advertising and physical capital investment decisions to maximize the value of the firm for shareholders. Advertising expenditures create brand capital which is a productive asset because it increases consumers' willingness to pay for the firm's good.

There are two frictions in the model: adjustment costs for both brand capital and physical capital, as well as costs of issuing new equity (external finance). The costs of adjusting physical capital and issuing new equity are standard. In addition, changing the stock of brand capital is costly because the planning and execution of advertising activities (even if outsourced) takes away resources from the firm's other productive activities, and is also typically associated with promotions and discounts. At the same time, it is difficult for firms to downsize by selling their brand(s) names. Because brand capital is costly to adjust, installed brand capital contributes to the firm's total market value.

Our analysis shows that the investment-based model augmented with brand capital replicates well both the asset pricing facts and key properties of brand capital and physical capital investment rates with reasonable parameter values. This result depends crucially on the existence of brand capital adjustment costs. Without these costs, brand capital investment (advertising) is too volatile (329% in the model versus 37% in the data). In addition, the average return spreads are tiny when compared with those in the data across both the brand capital investment portfolios (1.8% in the model versus 5.2% in the data), and the brand capital intensity portfolios (1.5% in the model versus 5.1% in the data).

In the model, the negative relationship between firms' brand capital investment rate (advertising expenditures) and future stock returns arises endogenously in the cross section due to differences in firms' productivity, and is amplified by the costs of adjusting the capital stocks. Investment is procyclical, and thus high productivity firms advertise and invest more than low productivity firms. In addition, high productivity firms produce relatively more output and have higher profits, and thus are less affected by adjustment costs. This gives these firms more flexibility to adjust their effective cash flows to shareholders over time in response to aggregate shocks. As a result, the value of high productivity firms is affected relatively less than the value of low productivity firms after large negative productivity shocks, that is, in bad economic times when the price of risk is high. The high productivity/high advertising firms are thus less risky and hence have low expected stock returns in equilibrium, consistent with the empirical results reported here. This mechanism is also consistent with the empirical findings in Imrohoroglu and Tuzel (2012), who show that more productive firms earn lower average stock returns in the U.S. economy.

Analogously, brand capital intensive firms are low productivity firms with a stock of brand capital that is too large relative to their labor force. These firms want to reduce their size because of their lower productivity, but the irreversibility of the brand capital stock, together with the cost of reducing the stock of physical capital, makes it costly to downsize. Because of the reduced ability of these firms to adjust their effective cash flows to shareholders over time in response to aggregate shocks, these firms are more risky and thus have higher expected returns in equilibrium.

[1] See, for example, Bagwell (2007) for a detailed discussion of alternative economic explanations for why consumers respond to advertising.

Finally, we show that the model rationalizes the pattern of stock returns and advertising expenditures around SEOs previously documented in the asset pricing literature.[2] In the data, advertising expenditures and stock returns are contemporaneously positively correlated, but future stock returns are negatively correlated with current advertising expenditures. In addition, advertising expenditures and stock returns tend to be unusually high in years in which firms issue new equity. These links are motivated by Merton's (1987) seminal work, which shows that firm value increases with the degree of investor recognition of the firm if investors only invest in securities that they know about. Thus, these empirical links are usually interpreted as evidence of imperfections in financial markets (e.g. limited attention by investors), and consistent with the hypothesis that managers use advertising expenditures to maximize short-term stock prices by increasing investors' attention to the firm.

The firm's optimal response of investment to productivity shocks in the model endogenously replicates the patterns observed in the data, thus providing a possible alternative explanation for the previous findings. In the model, external finance is procyclical: when facing a large increase in productivity, firms raise advertising expenditures and physical capital investment and simultaneously issue new equity to finance investment. At the same time, because of the large increase in productivity, realized stock returns are also high and expected stock returns are low due to the low risk of these firms. In short, in the model, what drives equity issuance is the firm's desire to grow which is driven by high productivity. It is thus natural that firms that want to grow also want to invest and advertise, consistent with the high contemporaneous correlation between SEOs and both advertising expenditures and stock returns.

1.1. Related literature

Our work is related to several strands of literature. Previous studies have investigated the link between intangible capital and asset prices.[3] Intangible capital is, however, a broad concept: there is heterogeneity in the different intangible capital assets. Thus, investigating the properties of different types of intangible capital is useful to better understand the impact of intangible capital on asset prices. For example, both the firm's stock of human capital skills and brand name contribute to the firm's total stock of intangible capital, but these two assets are likely to have different risk properties. Human capital skills are partially embodied in the firm's labor force. Because this stock of human capital is not fully owned by the firm, shareholders are exposed to the risk that a worker may leave a firm, and hence decrease the firm's stock of human capital skills. In contrast, a firm's brand name is firm specific because the firm fully owns the property rights to its brands.

The closest paper to ours is Eisfeldt and Papanikolaou (2013) who show that firms with more organizational capital are riskier than firms with less organizational capital. We investigate a different form of intangible capital. In addition, we interpret our empirical findings within the standard neoclassical investment-based model, thus emphasizing the role of adjustment costs in intangible capital in explaining the risk properties of brand capital. We also study the link between financing decisions and intangible capital.

Gourio and Rudanko (2011) study the implications of customer capital for firm value and investment dynamics. Like our study, they emphasize the importance of adjustment costs in creating customer capital. Brand capital and customer capital are related because both measures capture the effect of customer loyalty on firms' performance. Our analysis is different because we investigate the impact of brand capital on firms' risk.

Vitorino (forthcoming) uses the measure of brand capital proposed here, and investigates the impact of brand capital on asset returns through structural estimation. Our work differs because we consider a fully specified economy in which prices and quantities are endogenously determined. Through calibration and simulation, our focus is thus on understanding the economic determinants of the endogenous risk premiums associated with brand capital stock.

The theoretical approach in this paper is related to the literature on investment-based asset pricing.[4] The study of the link between advertising expenditures, stock returns and SEOs is similar in spirit to the analysis in Carlson et al. (2006), and Li et al. (2009), who also emphasize the importance of investment for understanding the conditional dynamics of expected stock returns around SEOs. Our empirical findings are related to the empirical asset pricing literature exploring the effect of firm characteristics on the cross section of stock returns. Fama and French (2008) provide a recent survey of the literature. We show that brand capital stock is a firm characteristic that is related to firm risk in the cross section.

[2] A non-exhaustive list of recent studies documenting these links include Chemmanur and Yan (2009) and Lou (2011) (see also Lehavy and Sloan, 2007 (and references therein), for a survey of the related literature). In related work, Grullon et al. (2004), show that advertising spending is positively associated with various liquidity measures. We do not examine liquidity in our model.

[3] A non-comprehensive list of related papers performing an analysis at the aggregate level include: Hall (2001), McGrattan and Prescott (2000), as well as Hsu (2009), who show that technological innovations forecast stock excess returns using R&D data, a form of investment in intangible capital. Studies in the cross section include: Hansen et al. (2004) who study the risk characteristics of intangible capital; Chan et al. (2001) who document a positive relation between R&D intensity and firms' future stock returns and Li (2011) who shows that this positive relation is only present in R&D intensive firms; Lin (2012) who explains the link between R&D expenditures and asset prices in a theoretical model; and Li and Liu (2010) who study the importance of intangible capital in a Q-theory model using structural estimation. Our work differs because we focus on a different measure of intangible capital, brand capital.

[4] A non-exhaustive list of studies in the investment-based literature include Cochrane (1991), Jermann (1998), Zhang (2005), Li et al. (2009), Tuzel (2010), Bazdresch et al. (2012), Lin (2012), Imrohoroglu and Tuzel (2012), Belo and Lin (2012), among others. See also Abel (1982), Hayashi (1982) and Summers (1981) for important early contributions in the Q-theory of investment that provided the foundations for the investment-based asset pricing literature initiated by Cochrane (1991). Our model is in the spirit of Griliches (1979) who incorporates intangible capital (R&D capital) into the firm's production function.

Finally, the empirical analysis is also related to the industrial organization as well as to the marketing literatures, which document strong correlations between advertising expenditures and stock returns.[5] In the absence of a theoretical model explicitly linking these variables in a setup in which these variables are endogenously determined, the observed correlations are difficult, if not impossible, to interpret. Our work attempts to fill this gap in the literature.

The paper proceeds as follows. Section 2 proposes a measure of brand capital stock, reports the empirical links between brand capital and asset prices, and revisits the link between advertising expenditures and stock returns around SEOs. Section 3 presents an investment-based model with brand capital that is used to interpret the empirical evidence. Section 4 calibrates the model and discusses the properties of the model solution. Section 5 provides a quantitative evaluation of the ability of the model to replicate the empirical findings, and performs several analyses to explain the economic forces driving the fit of the model. Finally, Section 6 concludes. A separate Appendix B with additional results and robustness checks is posted online.

2. Empirical findings

In this section, we show the empirical links between brand capital and stock returns in the cross section. We use the results reported here to motivate the neoclassical investment-based asset pricing model augmented with brand capital that we present in Section 3. To facilitate the comparison between some of the empirical results and the model results, we present the two sets of results (data versus model) side by side. Here, we focus on the analysis of the results in the data.

2.1. Data

Monthly stock returns are from the Center for Research in Security Prices (CRSP) and accounting information is from the CRSP/Compustat Merged Database Annual Industrial Files. We focus on firms listed on the NYSE, Amex, and Nasdaq. The sample is from 1975 to 2010 (before 1975 most firms do not report advertising expenditures). Only firms with ordinary common equity are included, meaning that ADRs, REITs, and units of beneficial interest are excluded. We also exclude from the sample firm-year observations with missing advertising expenditure data or for which the number of employees, gross capital stock, or book equity are missing or are negative. In addition, we omit firms whose primary SIC classification is between 4900 and 4999 (regulated firms), or between 6000 and 6999 (financial firms). Following Eisfeldt and Papanikolaou (2013), we require a firm to have a December fiscal year-end in order to align the accounting data across firms.

The main variables in our study are the firm's brand capital stock, and investment in this stock. These variables are constructed as follows. Investment in brand capital is given by advertising expenditures (A_t), and corresponds to Compustat data item XAD (advertising expenses). This variable is defined as the cost of advertising media (radio, television, periodicals, etc.) and promotional expenses. As discussed in Simon and Sullivan (1993), advertising affects firms' brand awareness through brand associations, perceived quality, and use experience. For example, advertising that provides information about verifiable attributes influences brand associations. Heavy advertising can enhance perceived quality of experience goods, that is goods whose quality cannot be determined prior to their purchase. Naturally, advertising expenditure data does not fully capture all the investments made by a firm to develop its brand (e.g. consistent product experience) and so our measure is an imperfect proxy for investment in brand capital. We are trading off this cost with the benefit that advertising expenditures accounting data is readily available for a reasonably large sample of firms and over a long period of time. This allows us to perform a broad cross-sectional and time series analysis thus helping to increase the power of our empirical tests.

To measure the stock of brand capital (B_t), we follow the literature on intangible capital and construct the stock of brand capital from past advertising expenditures data using the perpetual inventory method[6]:

$$B_t = (1-\delta)B_{t-1} + A_t. \tag{1}$$

To implement the law of motion in Eq. (1) we must choose an initial stock and a depreciation rate. According to the perpetual inventory method, we choose the initial stock as:

$$B_0 = \frac{A_0}{g+\delta},$$

where A_0 is the firm's advertising expenditure in the first year in the sample. We use a depreciation rate of $\delta = 50\%$ and an average growth rate of advertising expenditures of $g = 10\%$, which corresponds to the average growth rate in our sample. The seemingly large value for the brand capital depreciation rate is consistent with the empirical evidence surveyed in

[5] See Schmalensee (1972) and Bagwell (2007) for surveys of the economic analysis of advertising. Srivastava et al. (2006) and Srinivasan and Hanssens (2009) review the literature in marketing studying the link between marketing activities (including advertising expenditures), firm value and stock returns.

[6] The use of advertising data to construct the stock of brand capital is consistent with the approach in Doraszelski and Markovich (2007). Our empirical approach follows closely Eisfeldt and Papanikolaou (2013) (on organization capital). See also Hirschey and Weygandt (1985), Lev (2001), Lev and Radhakrishnan (2004), and Li and Liu (2010) for similar approaches. The Bureau of Economic Analysis uses a similar methodology to construct a stock of Research and Development capital using the item R&D expenses (see Sliker, 2007).

Bagwell (2007). For example, Lambin (1976, p. 96) reports that the depreciation rate estimates for advertising effects are on average around 50% per year across a series of products (our key asset pricing results are robust to using other depreciation rates, e.g., $\delta = 20\%$).

The brand capital investment rate is then given by the ratio of advertising expenditures to the beginning of the period brand capital stock ($IBK_t = A_t/B_{t-1}$). Observations with missing advertising expenditures are dropped from the sample. This leaves us with a sample with 22,744 firm-year observations.

In addition to brand capital, we use the following variables in the empirical work. Firm level capital investment (I_t) is given by Compustat data item CAPEX (capital expenditures) minus data item SPPE (sales of property plant and equipment). The capital stock (K_t) is given by the data item PPEGT (property, plant and equipment). The physical capital investment rate is then given by the ratio of physical investment to the beginning of the period capital stock ($IK_t = I_t/K_{t-1}$). Firms' sales are given by data item SALE (aggregate sales are the sum of firm-level sales). The number of employees (L_t) is given by data item EMP. Firms' hiring rate (HR) is given by the ratio of the change in the number of employees from year $t-1$ to year t. Market equity (Size) is price times shares outstanding at the end of December of t, from CRSP. Firm physical capital-to-market ratio (KM) is the ratio of the stock of physical capital to market equity. Leverage (LEV) is given by the ratio of assets minus book equity to the sum of market value of equity and assets minus book equity. Return on assets (ROA) is the ratio of data item NI (net income) and data item AT (book value of assets). Net stock issues is the difference from year $t-1$ to year t in the natural log of shares outstanding times an adjustment factor at $t-1$ and t.

2.2. Brand capital and asset prices

To study the link between brand capital and stock returns we construct five portfolios sorted on the firm's brand capital investment rate, and study the properties of post formation average excess stock returns and other characteristics of the portfolios. We focus on the link between investment in brand capital and stock returns by analogy to the approach in the large q-theory of investment literature which focuses on the link between physical capital investment and stock returns (see references in the Introduction section). Throughout this paper, we focus our analysis mostly on the properties of these portfolios. To provide a more comprehensive characterization of the link between brand capital and stock returns, we also consider an alternative sorting based on brand capital intensity.

We form the five investment in brand capital (IBK) portfolios as follows. In June of year t, we sort the universe of common stocks into five equally-sized portfolios based on the firm's brand capital investment rate at the end of year $t-1$. Once the portfolios are formed, their returns are tracked from July of year t to June of year $t+1$. The procedure is repeated in June of year $t+1$. In computing the returns of the portfolios, we report a modified version of value-weighted returns. We focus on value-weighted returns because of the low transaction costs associated with this procedure. Pure value-weighted portfolios are not necessarily well-diversified portfolios, because of the heavy tails of the size distribution in the U.S. stock market. As discussed in Fama and French (2008), the characteristics of value-weighted portfolios are dominated by a small number of very large firms. Thus, to characterize the link between brand capital and stock returns for a typical firm in the economy (and to be consistent with the theoretical model that we present below which is a model of the average firm in the economy), we impose a cap of 10% on the maximum weight of each firm in the portfolio at the time of portfolio formation. This choice guarantees that the minimum effective number of firms in each portfolio is ten.[7]

Panel A of Table 1 (columns under Data) reports our main empirical finding. The table reports the time-series averages of the IBK portfolio excess stock returns (r^s), and the corresponding t-statistics. The average excess returns of the IBK portfolios are decreasing in the brand capital investment rate. The spread in the average excess returns of these portfolios is large: the difference between the returns of the high and low IBK portfolios is −5.2% per annum, which is more than 2 standard errors from zero. This result is analogous to the well-established negative correlation between physical capital investment rates and future stock returns documented in previous studies (Cochrane, 1991) suggesting that, like physical capital investment, firms' advertising investment responds significantly to changes in risk premiums.[8]

Table 1 also reports the time series averages of the portfolio-level accounting variables measured at the time of portfolio formation (we use the within portfolio cross-sectional median across firms as the measure of portfolio level characteristic in each year). Firms in the low IBK portfolio relative to firms in the high IBK portfolio have lower physical capital investment rates (0.09 versus 0.21), lower hiring rates (−0.02 versus 0.13), and higher physical capital-to-market equity ratios (0.63 versus 0.38), and are slightly smaller (3.80 versus 4.57), and less profitable (0.00 versus 0.04).

[7] Our modified average value-weighted portfolio returns are effectively an intermediate case between equal-weighted and pure value-weighted average returns. In the data, the links between brand capital and returns are stronger in small firms than in big firms. Thus, the link between brand capital and stock returns in equal-weighted portfolios is stronger than the links reported here, but the link between brand capital and stock returns in pure value-weighted portfolios is weaker than the links reported here.

[8] The results are robust to the construction of the brand capital stock measure. We obtain similar results to those reported here when we sort on firms' advertising growth expenditures (following Lou, 2011), a variable that, in contrast with the IBK rate, does not require the measurement of brand capital stock (see online appendix).

Table 1
Brand capital and asset prices.

Portfolio	Data						Model					
	Low	2	3	4	High	H − L	Low	2	3	4	High	H − L
	Panel A: Investment in brand capital (IBK) portfolios											
r^s	8.39	7.61	6.42	5.55	3.17	−5.22	8.66	8.62	7.63	5.44	5.81	−2.85
[t]	2.30	3.06	2.15	1.82	0.83	−2.01	3.43	3.38	3.28	3.02	2.99	−3.44
IBK	0.35	0.51	0.60	0.72	1.20	0.85	0.15	0.35	0.49	0.59	1.01	0.86
IK	0.09	0.10	0.12	0.16	0.21	0.12	−0.03	0.00	0.10	0.16	0.40	0.43
HR	−0.02	0.00	0.03	0.06	0.13	0.15	−0.17	−0.09	−0.01	0.04	0.28	0.45
KM	0.63	0.64	0.55	0.48	0.38	−0.25	0.58	0.43	0.30	0.28	0.22	−0.36
Size	3.80	4.89	5.26	5.09	4.57	0.77	2.72	3.09	2.92	3.64	3.86	1.14
Lev	0.39	0.39	0.35	0.33	0.30	−0.09	–	–	–	–	–	–
ROA	0.00	0.04	0.05	0.06	0.04	0.04	0.30	1.00	1.01	2.11	4.14	3.84
	Panel B: Brand capital to labor ratio (BKL) portfolios											
r^s	2.60	6.05	4.88	5.99	7.71	5.11	4.93	5.96	9.19	7.70	11.44	6.51
[t]	0.71	1.80	1.57	2.03	2.91	1.92	2.89	3.12	3.32	3.27	3.74	4.36

This table reports the time-series averages of the characteristics of five portfolios sorted on investment in brand capital (IBK) (Panel A), and five portfolios sorted on brand capital to labor ratio (Panel B). The table reports the portfolio level (modified) value-weighted average excess returns (per annum, in percentage, and in excess of the risk free rate) denoted as r^s, with the corresponding heteroscedasticity-and-autocorrelation-consistent t-statistics, denoted as [t]. Panel A also reports the time series averages of the following accounting variables measured at the time of the portfolio formation: IK, the physical capital investment rate; IBK, the brand capital investment rate; HR, the hiring rate; KM, the physical capital stock-to-market equity ratio; Size, the log of market value; Lev, the leverage ratio; and ROA, the return-on-assets (in the model, ROA is measured as the exponent of the firm-specific productivity level z). In each point in time, we use the median across firms within a portfolio as the measure of the portfolio's characteristic. The left panels report the summary statistics of the variables in the data, for the period from 1975 to 2010. The right panels report the summary statistics computed as averages from 100 samples of simulated data from the theoretical model, each with 1000 firms and 50 annual observations. The model is simulated at the monthly frequency, and the accounting data is aggregated into annual.

We also examine an alternative set of brand capital portfolios. Specifically, we construct five portfolios sorted on firms' brand capital-to-labor (BKL) ratio.[9] This sorting procedure allows us to investigate the variation in risk associated with firms with different brand capital intensities. Panel B of Table 1 shows that more brand capital intensive firms have higher average excess returns than less brand capital intensive firms (the relationship is not perfectly monotone due to the high returns of portfolio 2). The difference in average returns of the high and low BKL portfolios is 5.1%, which is more than 1.9 standard errors from zero. Because both organizational capital and brand capital are intangible assets, this result is consistent with the findings in Eisfeldt and Papanikolaou (2013) who first show that organizational capital intensive firms (where organizational capital is scaled by book-value of assets) have higher average returns than less organizational capital intensive firms.

2.3. Inspecting the mechanism

The previous section documents the links between brand capital and average stock returns. In this section, we investigate the extent to which these links can be related to differences in the risk of firms with different rates of investment in brand capital. Understanding the driving forces of the link between brand capital and stock returns is important to understand the class of models that can potentially explain the links in the data.

2.3.1. Asset pricing tests

We investigate if the spread in the average returns across the IBK portfolios reflects a compensation for risk using the unconditional and conditional versions of the capital asset pricing model (CAPM) as the benchmark asset pricing models. To test the unconditional CAPM, we run time series regressions of the monthly excess returns of the IBK portfolios on a constant and on the excess returns of the market portfolio, and examine the size of the intercepts in the time series regressions (α, abnormal returns). To test the conditional CAPM, we follow Lewellen and Nagel (2006) and run CAPM regressions within each month using firm-level daily return data. We then report the average intercept (α^C) of these regressions for all portfolios across all months.

Table 2 reports the asset pricing test results. As reported in Panel A, the unconditional CAPM cannot explain the pattern in the returns of these portfolios. The mean absolute pricing errors (m.a.e.) are high (2% per annum). The pricing error of the high-minus-low IBK portfolio is also high, −6.2% per annum, and this value is more that 2.2 standard errors from zero. Panel B shows that the performance of the conditional CAPM is significantly better. The mean absolute pricing errors are 0.7% per annum, which is about a third of the mean absolute pricing error of the unconditional CAPM. In addition, the

[9] The sorting variable is $BKL_t = B_t/L_t$, in which B_t is the end of the period brand capital stock, and L_t is the end of the period number of employees. This variable is used at June of year $t+1$ to sort firms into five BKL portfolios using the same procedure as in the construction of the IBK portfolios (and a similar procedure to compute the modified value-weighted returns).

Table 2
Abnormal returns.

Portfolio	Low	2	3	4	High	H-L	m.a.e.
	Panel A: Unconditional CAPM						
α	1.91	2.65	0.82	−0.46	−4.27	−6.18	2.02
$[t]$	1.03	1.93	0.60	−0.39	−2.18	−2.27	
	Panel B: Conditional CAPM						
α^C	1.66	0.79	0.37	−0.16	−0.38	−2.04	0.67
$[t]$	2.60	1.58	0.68	−0.37	−0.49	−2.36	

This table reports the (modified) value-weighted average abnormal returns of the five portfolios sorted on investment in brand capital. H-L is the portfolio that is long in the high and short in the low investment in brand capital portfolio. α (in annual percent) is the regression intercept from unconditional monthly CAPM regressions (Panel A) of portfolio excess returns on a constant and the market portfolios. α^C is the average (in annual percent) regression intercept from conditional CAPM firm-level regressions performed every month (Panel B) using daily return data. $[t]$ are heteroscedasticity-and-autocorrelation-consistent t-statistics. m.a.e. is the mean (across portfolios) absolute error (α and α^C) in annual percent. The data is from 1975 to 2010.

pricing error of the high-minus-low IBK portfolio is −2% per annum. Although this value is still statistically different from zero, it is less than a third of the magnitude of the pricing error of this portfolio according to the unconditional CAPM. Thus, we can conclude that a conditional version of the CAPM explains a reasonable fraction of the returns of the IBK portfolios, consistent with the hypothesis that differences in risk are an important driving force behind the cross-sectional variation in the average returns of these portfolios.

2.3.2. Profitability regressions

We provide additional evidence on the higher risk of low IBK firms that is not based on asset prices data. We investigate whether there are systematic differences in the sensitivity of low and high IBK firms' operating performance to aggregate total factor productivity (TFP) shocks in the economy (see Imrohoroglu and Tuzel, 2012, for a similar approach). If low IBK firms are riskier than high IBK firms, then the operating performance of low IBK firms should be more sensitive to TFP shocks (i.e. more cyclical) than the operating performance of high IBK firms. Because the price of risk of TFP shocks is positive in most equilibrium models (e.g. Jermann, 1998), firms that covary more with the TFP shocks are riskier because these firms perform poorly when aggregate TFP is low, that is, in bad economic times.

We measure firms' performance using change in firms' log earnings ($\Delta \mathrm{EAR}_{it}$), in which earnings are measured as earnings before interest, taxes, depreciation and amortization (EBITDA).[10] We then run a time series regression of the form:

$$\Delta \mathrm{EAR}_{it} = \mathrm{a} + \mathrm{b} \times \Delta \mathrm{TFP}_t + \mathrm{c} \times \mathrm{D}_{it}(\mathrm{Low}) \times \Delta \mathrm{TFP}_t + \varepsilon_{it} \tag{2}$$

in which Δ is the first difference operator, $\Delta \mathrm{TFP}_t$ is the change in aggregate total factor productivity, and $\mathrm{D}_i(\mathrm{Low})$ is a dummy variable equal to one if firm i belongs to the low IBK portfolio in period $t-1$. The TFP data is from John Fernald's (Federal Reserve Bank of San Francisco) webpage. The regression includes firm fixed effects (specifically, all variables are demeaned within each firm), and the standard errors are robust to heteroscedasticity and are clustered by firm and year.

Eq. (2) is estimated on a sample of firms that includes only low and high IBK firms, that is, firms that belong to the low and high IBK portfolio at the end of year $t-1$. The slope coefficient associated with aggregate TFP in the previous regression thus captures the average sensitivity of a high IBK firm to the aggregate shock *after* the portfolio formation. This timing is thus consistent with the analysis of the ex-post average stock returns in the portfolio sorting procedure. The interaction between aggregate TFP and the dummy variable $\mathrm{D}_{it}(\mathrm{Low})$ allows us to perform a formal test of the differential sensitivity of the low and high IBK firms to the aggregate TFP shock.

The results (not tabulated) show that low IBK firms have a higher sensitivity to the aggregate TFP shock than high IBK firms. The slope coefficient associated with aggregate TFP is positive, 0.03, and this value is more than 3.4 standard errors from zero. The slope coefficient associated with the interaction between aggregate TFP and the dummy variable $\mathrm{D}_{it}(\mathrm{Low})$ is also positive, 0.02, and this value is about 1.8 standard errors from zero. Thus, the sensitivity of the earnings of low IBK firms to the aggregate TFP shock is significantly higher than the sensitivity of high IBK firms (0.05 versus 0.03). This result suggests that low IBK firms are riskier because they fluctuate more closely with economic conditions than high IBK firms. Together with the results from the conditional CAPM, the analysis in this section is consistent with the hypothesis that the higher average returns of low IBK firms relative to high IBK firms reflects a compensation for the higher macroeconomic risk of the low IBK firms. We formalize this hypothesis in the theoretical model next.

[10] Because we use log earnings, observations with negative earnings are dropped (about 16% of the observations). Using alternative measures of operating performance such as the change in firms' sales, the change in firms' return on assets, or changes in firms' return on equity, produces similar results to those reported here.

Table 3
Advertising and return dynamics around SEOs.

	Data				Model			
	IBK(t)		Ret(t)		IBK(t)		Ret(t)	
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
MB($t-1$)		0.10		−0.22		0.55		−0.19
[t]		4.22		−12.89		27.49		−5.56
Size($t-1$)		−0.16		0.06		0.03		0.01
[t]		−4.55		2.68		3.23		1.51
Event(t)	0.05	0.05	0.13	0.16	0.39	0.31	0.26	0.29
[t]	5.50	5.05	8.33	8.58	25.24	22.01	13.54	15.56
Pre-Event(t)	0.01	0.01	0.10	0.11	−0.08	0.01	0.06	0.05
[t]	0.76	0.70	10.35	10.94	−4.08	0.96	4.60	4.36
Post-Event(t)	0.02	0.03	−0.07	−0.04	0.22	0.02	−0.12	−0.06
[t]	2.17	2.55	−7.27	−5.20	19.20	0.75	−7.76	−3.76

This table reports estimates of the slope coefficients from a regression of brand capital investment rates (columns (i), (ii), (v), and (vi)) and annual stock returns (columns (iii), (iv), (vii), and (viii)) on lagged firm characteristics and three event dummies. An event year is one if the firm has conducted a seasoned equity offering (SEO) in that year. The variables are defined as follows: IBK(t) is the brand capital investment rate in year t, Ret(t) is the annual cumulated excess stock return in year t (January to December), MB($t-1$) is the log of market equity-to-physical capital ratio in December of year $t-1$, Size($t-1$) is the firm log market capitalization in December of year $t-1$, Event(t) is an indicator variable, which equals one if t is an event year, and zero otherwise, Pre-Event(t) equals one if $t+1$ is an event year, and Post-Event(t) equals one if $t-1$ is an event year. The regression includes both firm and year fixed effects. t-statistics [t] are computed from standard errors that are robust to heteroscedasticity and are clustered by firm and year. All variables are standardized to have mean zero and unit variance. The "Data" columns report the regression results in the data, at annual frequency. The sample period is 1975 to 2010. The "Model" columns report the regression results computed as averages from 100 samples of simulated data from the theoretical model, each of which has 1000 firms and 50 annual observations.

2.4. Advertising and SEOs

Previous studies document empirical links not only between advertising expenditures and stock returns, but also between advertising expenditures and seasoned equity offerings (SEOs). For example, Chemmanur and Yan (2009) and Lou (2011) document that firms' advertising expenditures are unusually large during SEOs years. Similarly, stock returns tend to be unusually high during an SEOs year, but unusually low in the following year (see also Loughran and Ritter, 1995, for the evidence related to stock market underperformance following SEOs). In this section, we revisit some of the evidence on these empirical links. We then use these links as an additional set of moments for the evaluation of the theoretical model proposed below.

In previous studies, the motivation for examining the link between advertising, stock returns, and SEOs follows from Merton (1987), who shows that when investors only invest in securities that they know about, firm value is increasing in the degree of investor recognition of the firm. Examining the link between advertising, stock returns, and SEOs then follows naturally from Merton's (1987) analysis because advertising expenditures are a natural way for managers to attract investors' attention and thus possibly increase the revenues from an SEO.

Here, we perform an empirical analysis that is similar to the analysis performed in Lou (2011). Table 3 (columns below Data) reports the regression results from a pooled OLS regression of firm level brand capital investment rate (columns (i) and (ii)) as well as firms' annual returns (columns (iii) and (iv)) on lagged firm characteristics (size and market-to-book ratio) and various firm-specific event dummies.[11] Event(t) is a dummy variable equal to one if the firm has conducted at least one seasoned equity offering in that year (which we measure in Compustat data as the observations in which a firm's net stock issuance is greater than 2%). Then, Pre-Event(t) equals one if $t+1$ is an event year, and Post Event(t) equals to one if $t-1$ is an event year.

Consistent with the results reported in Lou (2011) (which uses the advertising growth variable instead of the closely related investment in brand capital variable), the pattern of the slope coefficients across the event dummies reported in column (i) shows that investment in brand capital tends to be significantly larger during event years relative to all other years. Column (iii) shows that stock returns tend to be significantly larger during the pre-event and, especially, the event years, but significantly lower in the post-event years (this last slope is related to the well documented firm underperformance following SEOs). Controlling for firm characteristics does not change the conclusions (columns (ii) and (iv)).

Lou (2011) provides an interesting interpretation for these empirical links. If some investors suffer from limited attention as discussed in Merton (1987) and/or other forms of behavioral biases in forming expectations, increased advertising expenditures can cause higher contemporaneous stock returns but lower subsequent returns. This mechanism occurs because advertising increases the buying pressure through higher visibility of the stocks without necessarily changing the supply side pressure (because sellers hold a small number of stocks, they are likely to suffer less from limited attention). In turn, managers can use advertising expenditures to maximize short-term prices and maximize the revenues from SEOs. Thus, the patterns observed in the data are interpreted as being consistent with the hypothesis that firm managers use advertising

[11] The firm-level brand capital investment rate, size, and book-to-market variables are winsorized at the top and bottom 1% to reduce the impact of outliers.

expenditures to attract investors' attention and thus manipulate short-term stock market prices to increase the revenues from the SEO to their (or existing shareholders) benefit.

3. An investment-based model with brand capital

We propose a dynamic investment-based model to understand the link between advertising and brand capital, asset prices, and SEOs.

The model is a standard neoclassical investment model augmented with an intangible asset, brand capital. The economy is composed of a large number of firms and each firm produces a differentiated good. Firms accumulate brand capital through advertising expenditures, and make optimal advertising and investment decisions to maximize firm value. Firms are all equity financed. Issuing equity is costly and is the only source of external finance. Including costly external finance in the model allows us to study the link between advertising expenditures and SEOs observed in the data in a quantitatively meaningful way.[12]

3.1. Technology

We focus on the optimal production decision problem of one firm in the economy (whenever possible, we suppress any firm-specific subscripts to save on notation). The firm uses capital inputs K_t, and labor inputs, L_t, to produce output, Y_t, according to the constant returns to scale technology:

$$Y_t = e^{x_t + z_t} K_t^{\alpha} L_t^{1-\alpha}, \tag{3}$$

where $\alpha > 0$, x_t is aggregate productivity, and z_t is the firm's idiosyncratic productivity. Aggregate productivity follows the process:

$$x_{t+1} = \bar{x}(1 - \rho_x) + \rho_x x_t + \sigma_x \varepsilon^x_{t+1}, \tag{4}$$

where ε^x_{t+1} is an independently and identically distributed (i.i.d.) standard normal shock. Idiosyncratic productivity follows the process:

$$z_{t+1} = \rho_z z_t + \sigma_z \varepsilon^z_{t+1}, \tag{5}$$

where ε^z_{t+1} is an i.i.d. standard normal shock that is uncorrelated across all firms in the economy, and ε^x_{t+1} is independent of ε^z_{t+1} for each firm. In the model, the aggregate productivity shock is the driving force of economic fluctuations and systematic risk, and the idiosyncratic productivity shock is the driving force of firm heterogeneity.

In every period t, the capital stock K_t depreciates at rate δ_k and is increased (or decreased) by gross investment I_t. The law of motion of the capital stock is given by

$$K_{t+1} = (1 - \delta_k) K_t + I_t, \quad 0 < \delta_k < 1. \tag{6}$$

When changing the capital stock, firms incur capital adjustment costs. These capital adjustment costs include planning and installation costs, costs of learning how to use the new equipment, or costs related with production being temporarily interrupted. These costs are specified by the following convex asymmetric quadratic adjustment cost function:

$$C^I(I_t, K_t) \equiv \frac{\theta_{kt}}{2} \left(\frac{I_t}{K_t}\right)^2 K_t, \tag{7}$$

where

$$\theta_{kt} \equiv \theta_k^+ . \varkappa\{I_t \geqslant 0\} + \theta_k^- . \varkappa\{I_t < 0\}. \tag{8}$$

Here, $\varkappa\{I_t \geqslant 0\}$ is an indicator function that equals one if the event described in $\{.\}$ is true and zero otherwise. The adjustment cost function is asymmetric, that is $\theta^- > \theta^+ > 0$, to capture the idea of costly reversibility in Zhang (2005).

In addition to physical capital, firms accumulate brand capital through advertising expenditures (A_t). Brand capital does not affect production directly, but affects operating profits by changing the consumer's willingness to pay for the firm's goods. As such, brand capital, like physical capital, can be interpreted as a factor of production in the operating profit function of the firm. The law of motion of brand capital is given by:

$$B_{t+1} = (1 - \delta_B) B_t + A_t, \tag{9}$$

in which δ_B is the depreciation rate of brand capital.

[12] As we show below, without external financing costs, firms issue equity too often in the model, which makes it difficult to make a quantitative analysis of the link between SEOs and advertising expenditures in the model.

Investment in brand capital is irreversible, that is $A_t \geqslant 0$, which means that brand capital cannot be disembodied from the firm. As discussed in Simon and Sullivan (1993), mergers, acquisitions and divestitures of major business lines are difficult because they are frequently accompanied by significant restructuring of the corporation and repositioning of its products. Similar to physical capital, firms incur adjustment costs when expanding the stock of brand capital.[13] These costs capture the notion that planning of advertising campaigns is costly and takes away resources (e.g. workers) from other productive activities. In addition, advertising expenditures may be associated with an increase in customer support, discounts, etc. Furthermore, low scale local advertising campaigns are less expensive than large scale national campaigns since the later require the use of professional advertising agencies whereas the former can usually be done in-house, thus suggesting that adjustment costs are increasing in advertising expenditures. These costs are specified by the following convex adjustment cost function:

$$C^A(A_t, B_t) \equiv \frac{\theta_b}{2}\left(\frac{A_t}{B_t}\right)^2 B_t, \tag{10}$$

where $\theta_b > 0$ is a constant.

3.2. Goods and labor markets

Firms have some degree of market power. Each firm faces a downward sloping demand curve given by:

$$P(B_t, Y_t) = B_t^{\gamma} Y_t^{-\frac{1}{\varepsilon}}, \quad \text{where } 0 \leqslant \gamma < 1 \text{ and } \varepsilon > 1, \tag{11}$$

in which ε is the demand elasticity of the good. In this specification, the stock of brand capital increases the consumers' willingness to pay for the firm's good. Thus, all else equal, a higher stock of brand capital allows firms to increase sales, but at a diminishing rate ($\gamma < 1$). The positive effect of brand capital on firms' sales is consistent with the empirical evidence surveyed in Bagwell (2007, Section 3.1.1) and Schmalensee (1972, Chapter 4).[14]

The supply of labor is perfectly elastic at the (stochastic) aggregate wage rate W_t (or labor cost per worker), which firms take as given. Following Bazdresch et al. (2012), the equilibrium wage rate W_t is assumed to be an increasing function of the demeaned aggregate productivity shock:

$$W_t = \lambda \exp\bigl(\omega(x_t - \bar{x})\bigr), \tag{12}$$

with $\lambda > 0$ and $0 < \omega \leqslant 1$.

3.3. Stochastic discount factor

Firms take as given the market-determined stochastic discount factor (henceforth SDF) $M_{t,t+1}$, used to value the cash-flows arriving in period $t+1$. We specify directly the SDF in the economy without explicitly modeling the consumer's problem. The SDF is given by:

$$\log M_{t,t+1} = \log\beta + \gamma_t(x_t - x_{t+1}), \tag{13}$$

$$\gamma_t = \gamma_0 + \gamma_1(x_t - \bar{x}). \tag{14}$$

The parameters $\{\beta, \gamma_0, \gamma_1\}$ are constants satisfying $1 > \beta > 0$, $\gamma_0 > 0$ and $\gamma_1 < 0$. In Eq. (14), γ_t is time varying and decreases in the demeaned aggregate productivity shock $(x_t - \bar{x})$, to capture the well documented countercyclical price of risk with $\gamma_1 < 0$. Allowing for variation in the price of risk is also motivated by the fact that, in the data, a conditional version of the CAPM captures better the cross-sectional variation in the returns of the IBK portfolios than the unconditional version of the CAPM.

According to this specification of the SDF, the risk-free rate ($R_{f,t}$) and the maximum Sharpe ratio (SR_t) in the economy are given by:

$$R_{f,t} = \frac{1}{E_t[M_{t,t+1}]} = \frac{1}{\beta} e^{-\gamma_t(1-\rho_x)(x_t - \bar{x}) - \frac{1}{2}\gamma_t^2\sigma_x^2}, \tag{15}$$

$$SR_t = \frac{\sigma_t[M_{t,t+1}]}{E_t[M_{t,t+1}]} = \sqrt{e^{\gamma_t^2\sigma_x^2} - 1}. \tag{16}$$

The SDF is in units of a numeraire consumption good with price normalized to one.

[13] See Gourio and Rudanko (2011), for a similar specification in the context of customer capital, which is similar in spirit to brand capital (i.e. both stock variables capture customer loyalty).

[14] Some of the empirical work establishing a link between advertising expenditures and sales include Lambin (1976), who find that brand advertising has a significant and positive effect on the brand's current sales and market share. A non-comprehensive list of other studies includes Palda (1964), Peles (1971), Telser (1962), and Nelson (1974).

3.4. Corporate policies

Define the firm's operating profit as:

$$\Pi_t = P(B_t, Y_t).Y_t - W_t L_t - f,$$

where f is a fixed operating cost. We normalize the price of the two investment goods (advertising and physical capital investment) to one (the relative price of the two investment goods is thus set to be constant). When the sum of physical capital investment, I_t, brand capital investment, A_t, and total adjustment costs, $C_t \equiv C^I(I_t, K_t) + C^A(A_t, B_t)$, exceeds internal funds, Π_t, the firm raises new equity capital, e_t, from external markets:

$$e_t \equiv \max\{0, A_t + I_t + C_t - \Pi_t\}.$$

As in Li et al. (2009), we assume that new equity is the only source of external finance. External equity is costly (Smith, 1977). To capture this effect, we follow Gomes (2001), Hennessy and Whited (2005) and Li et al. (2009), and assume that for each dollar of external equity raised, firms must pay proportional flotation costs. There are also fixed costs of financing. Thus, we parameterize the total financing-cost function as:

$$\Lambda_t \equiv \Lambda(e_t) = \lambda_0 \mathbf{1}_{\{e_t>0\}} + \lambda_1 e_t,$$

in which $\lambda_0 > 0$ captures the fixed costs, $\mathbf{1}_{\{e_t>0\}}$ is an indicator function that takes the value of one if the event described in $\{\cdot\}$ occurs, and $\lambda_1 e_t > 0$ captures the proportional costs.

When the sum of investment and adjustment costs is lower than internal funds, the firm pays the difference back to shareholders. The payout, D_t, is defined as

$$D_t = \max\{0, \Pi_t - A_t - I_t - C_t\}. \tag{17}$$

3.5. Firm's maximization problem

Define CF_t to be the effective cash flow accrued to shareholders (cash distributions minus the sum of external equity raised and the financing costs) which is given by

$$CF_t \equiv D_t - e_t - \Lambda(e_t) = \Pi_t - A_t - I_t - C_t - \Lambda(e_t). \tag{18}$$

Define the vector of state variables as $s_t = (x_t, z_t, K_t, B_t)$ and let $V(s_t)$ be the cum-dividend value of equity for the firm in period t. The firm makes advertising A_t, physical capital investment I_t, and labor L_t decisions to maximize its cum-dividend value by solving the problem:

$$V(s_t) = \max_{A_{t+j}, I_{t+j}, L_{t+j} j=0..\infty} \left\{ \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j} CF_{t+j} \right] \right\}, \tag{19}$$

subject to the law of motion of capital equations (6) and (9), as well as to Eq. (18). The operator $\mathbb{E}_t[.]$ represents the expectation over all states of nature given all the information available at time t.

In the model, risk and expected stock returns are determined endogenously along with the firm's optimal production decisions. To make the link explicit, we can evaluate the value function in Eq. (19) at the optimum,

$$V(s_t) = CF_t + \mathbb{E}_t\left[M_{t,t+1} V(s_{t+1})\right] \tag{20}$$

$$\Rightarrow 1 = \mathbb{E}_t\left[M_{t,t+1} R^s_{t+1}\right] \tag{21}$$

$$\Rightarrow \mathbb{E}_t\left[R^s_{t+1}\right] = R_{ft} - R_{ft} \times Cov_t\left[R^s_{t+1}, M_{t,t+1}\right] \tag{22}$$

where Eq. (20) is the Bellman equation for the value function, the Euler equation (21) follows from the standard formula for stock return $R^s_{t+1} = V(s_{t+1})/[V(s_t) - CF_t]$, and Eq. (22) follows from simple algebra using Eq. (21). According to Eq. (22), firms whose stock returns have a high negative covariance with the SDF (i.e. that provide low returns when the marginal utility of consumption is high), are risky, and thus the average stock returns of these firms must be high in equilibrium to compensate investors for bearing the risk of holding these assets.

4. Properties of the model solution

In this section, we explain the calibration of the model. The choice of the parameter values is based on the values reported in previous studies whenever possible, or by matching known aggregate asset prices moments, as well as key firm-level real quantity moments. We do not target cross-sectional asset prices moments related with brand capital and rather focus on these moments to evaluate the model. We also characterize the properties of the model solution implied by

Table 4
Calibration.

Parameter	Symbol	Value
Technology		
Elasticity of output with respect to capital	α	0.36
Elasticity of price with respect to brand capital	γ	0.30
Price elasticity of demand	ε	2
Rate of depreciation for physical capital	δ_k	0.12/12
Rate of depreciation for brand capital	δ_b	0.50/12
Upward physical capital adjustment cost	θ_k^+	6
Downward physical capital adjustment cost	θ_k^-	84
Upward brand capital adjustment cost	θ_b	18.5
Fixed production cost	f	0.039
Fixed external financing cost	λ_0	0.005
Proportional financing cost	λ_1	0.01
Stochastic processes		
Multiplicative coefficient on wage rate process	λ	0.02
Sensitivity of the wage rate to aggregate productivity	ω	1
Long-run mean aggregate productivity	$\bar{x}$	−4.4
Persistence coefficient of aggregate productivity	ρ_x	$0.98^{1/3}$
Conditional volatility of aggregate productivity	σ_x	0.004
Persistence coefficient of firm-specific productivity	ρ_z	0.97
Conditional volatility of firm-specific productivity	σ_z	0.20
Time-preference coefficient	β	0.999
Constant price of risk	γ_0	18
Time-varying price of risk	γ_1	−750

This table presents the calibrated parameter values used to simulate the baseline model.

the baseline calibration of the model, to help us understand the endogenous link between firm characteristics (advertising) and both stock returns (risk) and SEOs.

All the endogenous variables in the model are functions of the state variables. Because the functional forms are not available analytically, we solve for these functions numerically. To produce the moments implied by the model, we simulate 100 artificial panels, each of which with 1000 firms and 900 months and report the cross-panel average (and, in some cases, the median, and the 5th and 95th percentiles) moments. In each simulation of the model, we start by assuming the initial physical capital and brand capital stocks of all firms to be at their long-run average level and by drawing the firm-specific productivity levels from the unconditional distribution of z_t. We drop the first 300 months of data to neutralize the effect of the initial conditions. The remaining 600 months of data are treated as those from the stationary distribution. Appendix A provides a description of the solution algorithm and of the numerical implementation of the model.

4.1. Calibration

The model is calibrated at the monthly frequency using the parameter values reported in Table 4. The first set of parameters specifies the technology of the firm. The second set of parameters describes the exogenous stochastic processes that the firm faces, including the aggregate and idiosyncratic productivity shock, and the stochastic discount factor. Because the accounting variables in the real data are observed at annual frequency, the simulated accounting monthly data is aggregated to annual frequency.

4.1.1. Firm's technology

We set the elasticity of output with respect to capital in the production function to be $\alpha = 0.36$, similar to Cooley and Prescott (1995) and Gomes (2001). The elasticity of price with respect to brand capital γ is set at 0.3 to match the advertising expenditure to market equity ratio in the data as close as possible (4% in the data). We follow Caballero and Pindyck (1996) and Zhang (2005) and set the price elasticity of demand ε to be 2. The monthly capital depreciation rate δ_k is set at 1% per month as in Zhang (2005). The monthly depreciation rate of brand capital is set at $\delta_n = 4.16\%$ per month, which corresponds to an annual depreciation rate of 50%, consistent with the empirical procedure used to estimate the brand capital stock in the data.

Empirical estimates of the slope parameters of the capital adjustment cost function vary substantially across studies and so they are difficult to calibrate. In addition, there is no prior study examining the properties of brand capital adjustment costs. Our calibration strategy consists of matching firm-level real quantities as close as possible. We set the upward and downward parameters in the convex capital adjustment cost function at $\theta_k^+ = 6$ and $\theta_k^- = 84$ to match the volatility of the capital investment rate. Similarly, the slope adjustment cost of brand capital is set at $\theta_b = 18.5$ to match the volatility of the brand capital investment rate. The two parameters in the wage rate function are set at $\lambda = 0.02$, and $\omega = 1$, so that the volatility of the annual HP-filtered aggregate wages and the aggregate investment-to-wage bill ratio are as close to the data as possible. We set the fixed cost at $f = 0.039$ to match the value premium (i.e. the difference in average returns between the high and low decile portfolio sorted on firms' book-to-market ratio) in the data (5.2% per annum in our sample). We

Table 5
Selected moments.

Moment	Data	Model			
		Mean	5th	Median	95th
Asset prices					
Average risk-free rate (%)	1.80	0.96	−2.73	0.31	6.49
S.D. of risk-free rate (%)	3.00	1.34	0.46	1.20	2.49
Equity premium (%)	6.12	6.64	3.20	6.58	9.93
Value premium (%)	5.16	5.22	0.94	3.63	12.46
Market Sharpe ratio	0.39	0.46	0.19	0.38	0.98
Aggregate advertising-to-market equity	0.04	0.05	0.04	0.05	0.06
Input prices and real quantities					
S.D. of aggregate wages (%)	0.83	0.82	0.56	0.82	1.05
Aggregate investment-to-wage bill ratio	0.33	0.39	0.29	0.40	0.45
Average equity issuance	0.25	0.29	0.22	0.30	0.34
S.D. of physical investment (IK)	0.22	0.23	0.21	0.23	0.26
S.D. of brand capital investment (IBK)	0.37	0.39	0.38	0.39	0.41
Correlation of IK and Δ Sales Aggregate*	0.08	0.24	0.11	0.24	0.41
Correlation of IBK and Δ Sales Aggregate*	0.10	0.17	0.06	0.17	0.29

This table reports selected moments from the data and from the baseline calibration of the model. To obtain the model moments, we simulate 100 artificial panels, each of which has 1000 firms and 50 annual observations. For each moment in the model, we report the mean, the 5th, 50th (median), and 95th percentiles of the corresponding moment across simulations. The model is simulated at monthly frequency and the accounting data is aggregated into annual. Moments marked with an asterisk are not targeted in the calibration of the model. S.D. stands for standard deviation.

calibrate the fixed financing and proportional financing costs parameters (λ_0 and λ_1) to match the average equity issuance of 25%, reported in Li et al. (2009), as close as possible.[15]

4.1.2. *Stochastic processes*

The persistence ρ_x and conditional volatility σ_x of aggregate productivity are from King and Rebelo (1999) – $\rho_x = 0.98^{1/3}$, and $\sigma_x = 0.007/\sqrt{3} = 0.004$. The long-run average level of aggregate productivity, $\bar{x}$, is a scaling variable. We set the average long-run brand capital in the economy at three, which implies that the long-run average of aggregate productivity $\bar{x} = -4.4$. We calibrate persistence ρ_z and conditional volatility σ_z of firm-specific productivity to match the degree of dispersion in the cross-sectional distribution of firms' stock return volatilities. This procedure leads to $\rho_z = 0.97$ and $\sigma_z = 0.20$.

Following Zhang (2005), we pin down the three parameters governing the stochastic discount factor, β, γ_0, and γ_1, to match three aggregate return moments: the average real interest rate, the volatility of the real interest rate, and the average annual Sharpe ratio. This procedure yields $\beta = 0.999$, $\gamma_0 = 18$, and $\gamma_1 = -750$.

4.2. *Evaluation of the calibration*

To evaluate the quality of the calibration, we report moments for selected variables in the data and compare them with the corresponding moments generated in the model. To compute the model moments, we focus most of our analysis on the mean of each moment across the 100 simulated panels. For completeness, we also report the 5th, 50th (median), and 95th percentiles of each moment across simulations. Table 5 shows that the model generates moments that are comparable with those in the data.[16]

Focusing first on the fit along the asset prices dimension, Table 5 shows that the model generates a low (around 1%) and smooth (1.3% standard deviation) risk-free rate, and a high Sharpe ratio (0.46). The equity risk premium is also comparable to the data (6.6%) and the model generates a large value premium (5.2%) in the cross section. Finally, the average advertising-to-market equity ratio is in line with the data.

Turning to the analysis of real quantities and input prices, Table 5 shows that, in the data, the time series properties of investment in brand capital (IBK) and physical capital (IK) are comparable. Both the firm-level investment rate and, especially, the advertising rate are volatile (22% and 37% per annum for IK and IBK, respectively) and procyclical: the correlations of IK and IBK with aggregate sales growth are both positive (8% and 10% per annum, respectively). The analysis of the firm-level moments reported here is consistent with the analysis in Molinari and Turino (2009) for the aggregate level properties of advertising expenditures and physical capital investment in the U.S. economy (see their Tables 1 and 2).

[15] In the data, the frequency of equity issuance reported by the previous studies discussed in Li et al. (2009) (henceforth LLZ) is smaller than 25% (e.g. Hennessy and Whited, 2005, report an average of 9%). LLZ argue that an average of 25% or above is a more accurate number because the figures in most previous studies do not incorporate several forms by which firms issue equity. For example, Fama and French (2005) show that firms can issue equity in mergers and through private placements, convertible debt, warrants, direct purchase plans, rights issues, and employee options, grants, and benefit plans, all of which significantly increase the average of equity issuance.

[16] In computing the volatility of firm level investment rates (IBK and IK) in the data, we drop observations in which the investment rates are above 500% in a given year to reduce the influence of outliers.

The model is consistent with the properties of investment in brand capital and physical capital in the data. In the model (because there is no growth), the mean of both series is pinned down by the depreciation rate, so we don't report that value here. The volatility of the two investment series is very close to the data and, consistent with the data, both series are procyclical: the correlation with aggregate sales growth is 24% for IK and 17% for IBK (the calibration does not target these moments). Finally, the model matches reasonably well the properties of the aggregate wage rate and the average equity issuance.

4.3. The firm's value function, policy functions, and risk

To help understand the model, Fig. 1 plots the firm's value function, policy functions (gross physical capital investment, advertising expenditures, and new equity) and risk, as measured by the firm's conditional beta obtained from the numerical solution of the model. The firm's conditional beta is defined as $\beta_t = -Cov_t[R^s_{t+1}, M_{t,t+1}]/Var_t[M_{t,t+1}]$ which follows from a simple manipulation of Eq. (22). These functions depend on four state variables: the capital stock K_t, the brand capital stock B_t, the aggregate productivity x_t, and the idiosyncratic productivity z_t. Because the focus of this paper is on the cross sectional heterogeneity, we fix the aggregate productivity at its long-run average, $x_t = \bar{x}$, and we plot each function at two-different values of the idiosyncratic productivity z_t. The top panels in each figure plot each function against K_t and z_t, with B_t and x_t fixed at their long-run average levels $\bar{B}$ and $\bar{x}$. The bottom panels in each figure plot each function against B_t and z_t, with K_t and x_t fixed at their long-run average levels $\bar{K}$ and $\bar{x}$.

The top and bottom left panels in Fig. 1 show that the firm's market value is increasing in the firm's productivity, as well as in physical capital and brand capital stock. Thus, all else equal, firms with higher stocks of brand capital have higher market values.

According to the panels in the center of Fig. 1, gross investment and advertising expenditures are increasing in the firm's productivity. Thus, more productive firms invest and accumulate more brand capital than less productive firms, all else being equal. For investment, this result is consistent with the evidence documented in Fama and French (1995). Gross investment and advertising expenditures are also decreasing in capital stock and brand capital stock. Thus, small firms with less physical capital and brand capital, invest and accumulate more brand capital (and thus grow faster) than big firms with more physical capital and more brand capital. This result is consistent with the evidence provided by Hall (1987). Due to the irreversibility constraint on brand capital, firms with high brand capital stocks and low productivity have a large inaction region. Thus the model predicts that large firms with low productivity do not spend resources on advertising as these firms face a binding irreversibility constraint.

Fig. 1 also shows that equity tends to be issued by smaller firms. In addition, equity issuance is procyclical because, all else equal, more productive firms issue more equity than less productive firms, consistent with the analysis in Li et al. (2009). Combining the policy function for advertising expenditures with the policy function for new equity, we observe that firms that advertise more are more likely to issue equity, that is, advertising expenditures and equity issuance are positively correlated.

Finally, the plot of the conditional beta in the top and bottom right panels in Fig. 1 shows that firm's risk is decreasing in firm's productivity, as well as in the firm's level of physical capital and brand capital stocks. This result is consistent with the empirical findings in Imrohoroglu and Tuzel (2012), who show that more productive firms earn lower average stock returns in the U.S. economy, and is also consistent with Li et al. (2009) who show that small firms with less capital are more risky than big firms with more capital. The negative relationship between the firm's beta and the stock of brand capital is also consistent with the theoretical literature in marketing (e.g. Srivastava et al., 2006) who argue that investment in intangible assets such as brands, may enable firms to lower firm level risk and increase firm value.

5. Quantitative results

In this section, we replicate the empirical procedures on the simulated data and evaluate the ability of the model to match the main links between brand capital (advertising expenditures), stock returns, and SEOs as documented in the empirical section. We then explain the economic forces driving the results in the model. Finally, we report the results from two alternative calibrations of the model to help us understand the role of some of its key features, in particular, the properties of the firm's technology.

5.1. Brand capital and asset prices

The right columns in Panel A of Table 1 (columns "Model") report the average excess returns and the characteristics of the five IBK portfolios in the simulated data. The calibration of the baseline model generates a pattern of average excess returns across the IBK portfolios that is similar to the pattern in the data. After portfolio formation, firms in the low IBK portfolio have higher average returns than firms in the high IBK portfolio, a difference of 2.9% per annum. This difference in returns is large and statistically significant but is smaller than the 5.2% per annum difference in returns reported in the data. A potential explanation for the lower spread in the model relative to the data is the lack of financial leverage effects in the model. In the data, Panel A of Table 1 shows that low IBK firms have higher leverage ratios than high IBK firms

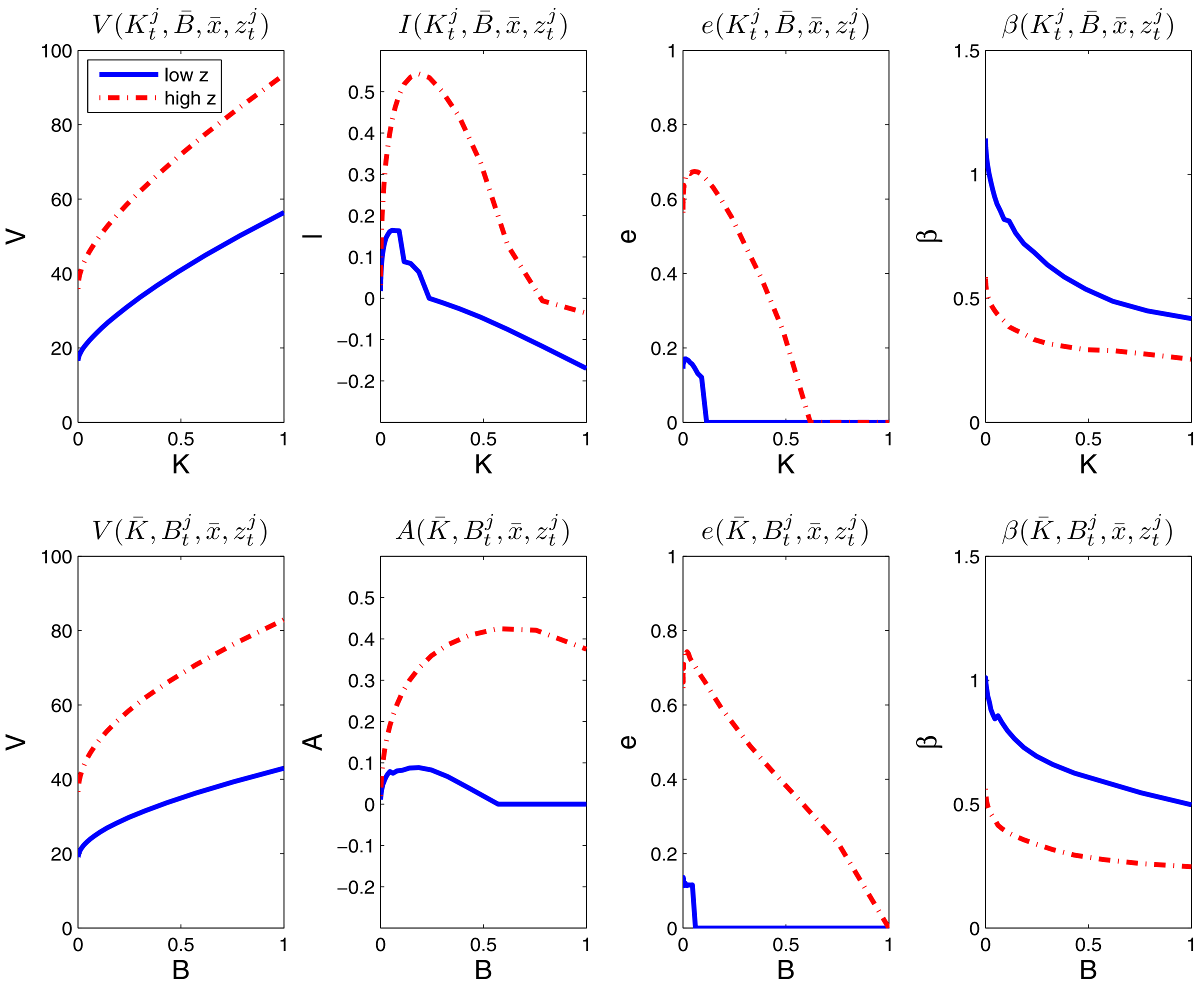

Fig. 1. Value function, policy functions, and conditional beta. For a given firm j, this figure plots the value function $V(K_t^j, B_t^j, x_t, z_t^j)$, the policy functions of physical capital investment $I(K_t^j, B_t^j, x_t, z_t^j)$, advertising expenditures $A(K_t^j, B_t^j, x_t, z_t^j)$, and equity issuance $e(K_t^j, B_t^j, x_t, z_t^j)$, as well as the firm's conditional beta $\beta(K_t^j, B_t^j, x_t, z_t^j)$, against the current level of physical capital stock (top panels) and brand capital stock (bottom panels). The firm's conditional beta is given by $\beta_t \equiv -Cov_t[R_{t+1}^s, M_{t,t+1}]Var_t[M_{t,t+1}]^{-1}$ which follows from Eq. (22). In the plots in the top panels, we fix the aggregate productivity x_t and brand capital B_t^j at their respective long-run average level of $\bar{x}$ and $\bar{B}$, and in the plots in the bottom panels, we fix the aggregate productivity x_t and physical capital K_t^j at their respective long-run average level of $\bar{x}$ and $\bar{K}$. The stocks of physical capital and brand capital are normalized to be between zero and one. Each panel reports two curves corresponding to a low firm-specific productivity z_t^j firm (solid line) and a high firm-specific productivity z_t^j firm (dashed line).

(0.39 versus 0.30), a characteristic that contributes to the higher average stock returns of the low IBK firms because stock returns in the data are levered.

The table also shows that the model successfully replicates the patterns of the characteristics of the IBK portfolios. The model generates a negative relationship between the physical capital-to-market equity (KM) ratio and the hiring (HR) and investment (IK) rates, as well as a positive relationship between productivity (which in the model is measured as the firm-specific productivity level, $\exp(z_t)$) and the brand capital investment rate.

Turning to the analysis of the five brand capital intensity (BKL) portfolios, Panel B in Table 1 reports the average excess returns of these portfolios. The model is qualitatively consistent with the difference in average returns of the BKL portfolios, although it slightly overshoots the magnitude of this spread: the BKL spread is 6.5% per annum in the model versus 5.1% in the data.

5.1.1. Interpretation

Why do firms with higher investment in brand capital investment rates have lower average returns (risk premium) in equilibrium? The characteristics of the portfolios reported in Table 1 provide suggestive evidence that helps to answer this question. The portfolio characteristics reveal that firms in the high IBK portfolio tend to be more productive than firms in the low IBK portfolio both in the data and in the model.[17] In turn, as reported in Fig. 1 (conditional beta), the high productivity firms have lower risk than the low productivity firms.

To illustrate why more productive firms have lower macroeconomic risk than less productive firms in the model, Fig. 2 shows impulse responses of selected endogenous variables in the baseline calibration of the model to a one-standard-deviation negative aggregate productivity shock. We report the responses of each variable in percentage deviation relative to their long-run average levels. Because all firms in the economy are ex-ante identical, we generate cross-sectional heterogeneity by examining the response of two firms in which their respective firm-specific productivity level is set at $z = +1.42$ (which we label as a high productivity firm) or at $z = -1.42$ (which we label as a low productivity firm) (these values of z correspond to the maximum and minimum values of firm-specific productivity on the grid); furthermore, their productivity levels gradually mean revert to the average level following the law of motion for z in Eq. (5). Specifically, we start with two firms with all state variables at the long-run average levels, and then change both the firm-specific productivity level and aggregate productivity level to reflect the shocks. The high and low productivity firms correspond roughly to the high and low IBK firms in the model. Even though the difference in productivity is not the only difference across these firms, it is clearly an important state variable.

Fig. 2 shows that, after the negative aggregate TFP shock, the high productivity firm has levels of investment and advertising that are significantly above the long-run mean level. This is because the firm's total (firm-specific plus aggregate) productivity is higher than its long-run average level, despite the negative aggregate TFP shock. Given its high productivity, this firm also has relatively high profits. For the low productivity firm, the fact that its current total productivity level is below the long-run average level, makes it optimal for this firm to downsize, and hence this firm has initial investment and advertising levels that are slightly below the long-run average levels. Thus, and despite the asymmetry in the adjustment cost function (which makes it more costly to disinvest than to invest), the total adjustment costs of the low productivity firm are smaller than those of the high productivity firm.

Due to the large initial and persistent increase in investment and advertising expenditures, as well as to the associated increase in adjustment costs, the high productivity firm issues more equity (not plotted) to finance both investments, and hence it decreases its effective cash flow (CF) accrued to shareholders in the first few periods after the aggregate shock (the low productivity firm is still able to increase its effective cash flow because it is investing and advertising less than the long-run average levels). With investments and adjustment costs mean-reverting to the long run average levels, the initial high investment and advertising levels of the high productivity firm start to pay off to shareholders as this firm eventually starts distributing higher cash flows to shareholders than the low productivity firm. These higher effective cash flows explain the relatively higher continuation value $(V(t+1))$ of the high productivity firm after the shock, because the continuation value is simply the present value of all future cash flows. In turn, because stock returns are mostly determined by the change in the continuation value (the standard capital gain component of stock returns is quantitatively more important than the dividend component captured here by CF), the relative pattern of the continuation values of these two firms after the aggregate shock explains why high productivity firms are relatively less risky: the returns of high productivity firms are relatively less affected by the negative aggregate shock (that is, bad economic times which correspond to periods of high marginal utility, SDF) than the returns of the low productivity firms. As a result, investors require a relatively smaller risk premium for holding shares of high productivity firms in equilibrium.

The exact same economic mechanism explains the higher macroeconomic risk of the firms with high brand capital intensity (BKL) firms. In both the data and the model, high BKL firms are less productive than low BKL firms. In the data, the high BKL firms have an ROA of 0.04 and the low BKL firms have an ROA of 0.05 (not tabulated). In the model, the average firm-specific productivity is 0.47 and 3.44 for the high and low BKL firms, respectively (not tabulated). To the extent that ROA is positively correlated with productivity, the model is consistent with the data. Taken together, the results

[17] In the data, we measure productivity using return on assets (ROA). We note, however, that the correlation between TFP and ROA is not very high, as reported in Imrohoroglu and Tuzel (2012). Thus, we acknowledge that ROA is an imperfect proxy of firm productivity.

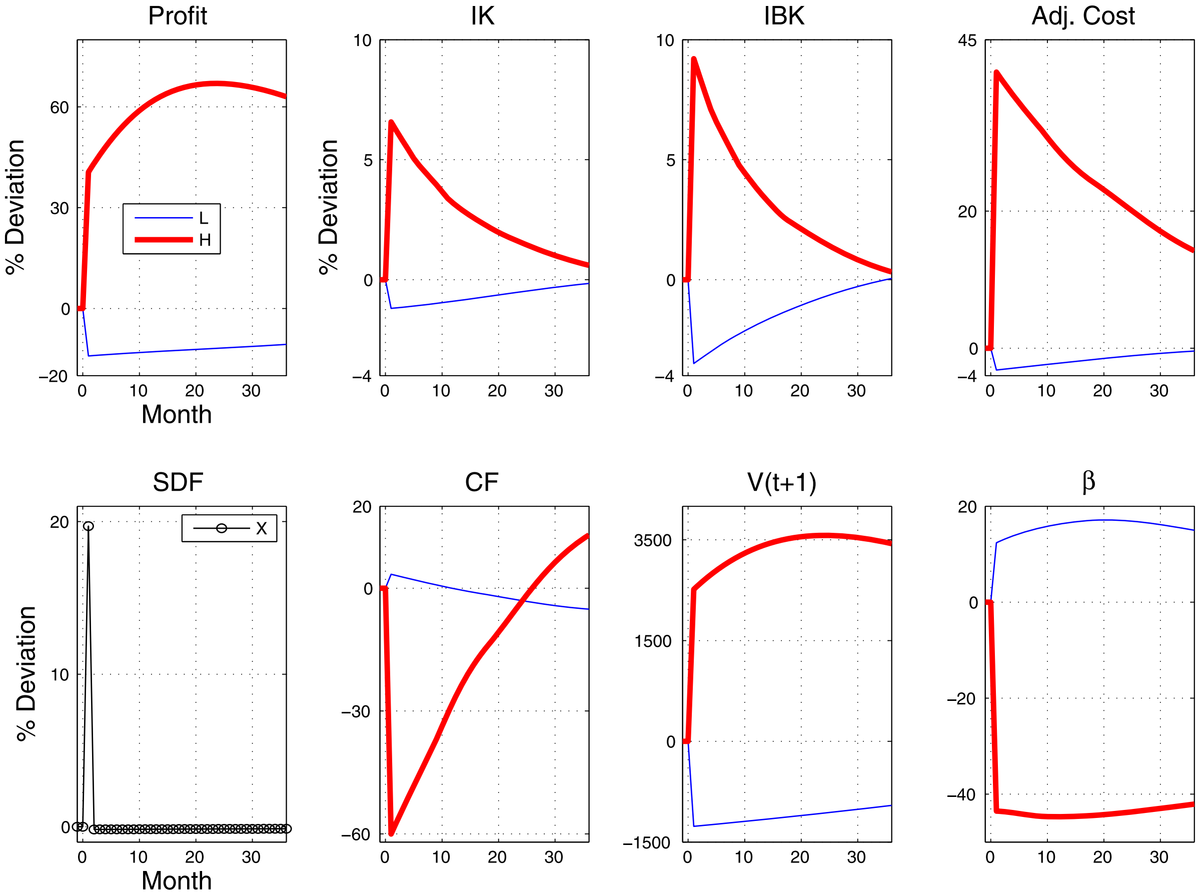

Fig. 2. Impulse responses to a negative aggregate productivity shock. Impulse responses of selected endogenous variables in the baseline calibration of the model to a one-standard-deviation negative aggregate productivity shock. The responses are measured in percent deviation relative to the long-run average values. To generate the response of a high productivity (H) firm and a low productivity (L) firm, we examine firms with firm-specific productivity at the highest and lowest points in the grid for the firm-specific shock (z). The frequency of the data is monthly. Profits is sales minus wage bill, IK is firms' physical capital investment rate, IBK is firms' brand capital investment rate (advertising), Adj. Cost is firms' total (physical and brand capital) adjustment costs, SDF is the stochastic discount factor (consumers' marginal utility), CF is firms' payout or effective cash flows accrued to shareholders (cash distributions minus the sum of external equity raised and the financing costs), and V is the continuation value of the firm (price of the firm after payout), β is firms' conditional beta given by Eq. (22).

show that differences in productivity are an important driving force of the link between advertising and stock returns observed in the data.

The link between brand capital intensity and average returns is consistent with the findings in Eisfeldt and Papanikolaou (2013) who first show that organizational capital (an alternative measure of intangible capital) intensive firms have higher average returns than less organizational capital intensive firms. The explanation proposed by our model is different. In Eisfeldt and Papanikolaou (2013) organizational capital is risky because it is partially embodied in firms' labor input and thus cannot be wholly owned by shareholders. This makes organizational capital risky because the division of firms' rents between labor and shareholders varies over time, exposing shareholders to an additional risk factor that is not perfectly correlated with aggregate TFP shocks. Our explanation is based on the neoclassical model of investment, in particular, it relies on the existence of adjustment costs in both brand and physical capital, and on the endogenous correlation between changes in firms' value and aggregate TFP shocks that follows from firms' optimal investment and advertising decisions. In our setup, brand capital is a risky asset because it is costly to adjust the stock of brand capital in response to aggregate shocks, thus affecting firms' ability to change the temporal distribution of the effective cash flow accrued to shareholders over time. As we show and discuss below, the existence of frictions in the accumulation of brand capital is crucial for the positive asset pricing results reported here in the context of a neoclassical investment model.

5.2. Advertising and SEOs

We now investigate the ability of the model to replicate the pattern of advertising expenditures and stock returns around SEOs documented in the empirical section.

Table 6
Comparative statics.

	Quantities			Asset prices				SEO regressions					
	S.D.			High-minus-low				IBK(t)			Ret(t)		
Spec.	IBK	IK	Issue	IBK	BKL	Value	A/ME	Evt	Pre	Post	Evt	Pre	Post
Data													
–	0.37	0.22	0.25	−5.22	5.11	5.16	0.04	0.05	0.01	0.03	0.16	0.11	−0.04
Baseline model													
1	0.39	0.23	0.29	−2.85	6.51	5.22	0.05	0.31	0.01	0.02	0.29	0.05	−0.06
Frictionless brand capital ($\theta_b = 0$, no irreversibility)													
2	3.29	0.28	0.52	−1.78	1.54	−0.43	0.03	0.23	0.03	−0.08	0.17	0.10	−0.05
Zero issuance costs ($\lambda_0 = \lambda_1 = 0$)													
3	0.39	0.23	0.47	−2.50	6.97	4.48	0.05	0.24	0.02	−0.02	0.20	0.02	−0.05

This table presents selected moments in the data and in the model, using simulated data generated by three alternative calibrations of the model (specifications 1 to 3). The table reports the following moments: Quantities – the standard deviation (S.D.) of firm-level brand capital investment rate (IBK) and physical capital investment rate (IK), and the average frequency of equity issuance (Issue); Asset prices – the average returns of the high-minus-low investment in brand capital (IBK) portfolio, the average returns of the high-minus-low brand capital to labor ratio (BKL) portfolio, and the average returns of the high-minus-low book-to-market (Value) portfolio; and SEO regressions – the slope coefficients on the event dummies from the SEO regressions reported in Table 3 using size and market-to-book ratio as control variables. The reported statistics in the model are averages from 100 samples of simulated data, each with 1000 firms and 50 annual observations.

The right panels in Table 3 (columns (v) to (viii)) report the results from the SEO regressions in the model. The investment-based model replicates the pattern of investment in brand capital and stock returns around SEOs reasonably well. Consistent with the empirical analysis, advertising expenditures tend to be particularly large during event years. As reported in columns (v) and (vi), the slope coefficients associated with the event dummy are both large and statistically significant. According to specification (vi), the slope coefficients associated with the pre-, and post-event dummies are both positive, consistent with the pattern in the data reported in column (ii), but they are significantly smaller than the slope coefficient associated with event dummy. Columns (vii) and (viii) shows that model generates a pattern of stock returns across the event dummies that mimics exactly the pattern of stock returns across event dummies estimated in the data.

5.2.1. Interpretation

The investment-based model replicates the pattern of investment in brand capital (advertising) and stock returns around SEOs. Thus, the model provides an alternative investment-based explanation for the empirical findings. The reason for the good fit is the following. In the model, on average, firms who issue stocks are those firms with better investment opportunities, that is, high productivity firms, consistent with the policy functions reported in Fig. 1. Their high productivity leads to high realized contemporaneous stock returns and to a positive correlation between advertising expenditures and stock issuance because: (i) firms want to invest more in brand capital to take advantage of the high marginal product of brand capital; and (ii) to finance the investment, firms issue new equity. In addition, because of their high productivity, these firms have low risk (see Fig. 1, right panels) and hence have low expected returns, which in turn explains the negative correlation between current advertising expenditures and future stock returns, especially for the firms that issue equity. In summary, in the model, what drives equity issuance is the desire to grow, which is driven by high productivity. It is thus natural that firms who want to grow, also want to invest and advertise, which generates a high contemporaneous correlation between SEOs and advertising expenditures and stock returns.

5.3. Alternative calibrations

To help us understand the role of some of the key features in the model, in particular, the properties of the firm's technology and the importance of brand capital adjustment costs, we report the results from two alternative calibrations. In each specification, we vary a set of parameters (mechanism) at a time, while keeping the other parameters equal to the baseline calibration. Thus, our analysis can be interpreted as a standard comparative statics exercise.

Table 6 reports selected moments generated by the alternative calibrations of the model. For comparison, in specification 1, we report the results in the baseline model. In specification 2, we examine the importance of brand capital adjustment costs in the model and consider a version of the model in which brand capital adjustment costs are set to zero ($\theta_b = 0$, and no irreversibility constraint). In specification 3, we set the costs of issuing equity to zero ($\lambda_0 = \lambda_1 = 0$).

The results in Table 6 (specification 2) show that brand capital adjustment costs are crucial for the investment-based model to be able to match the data, both on the quantity side and on the asset pricing side. In terms of real quantities, the table shows that without brand capital adjustment costs, the model counterfactually generates an investment in brand capital rate that is too volatile (329% here versus 39% in the baseline model and 37% in the data). This is expected because, without brand capital adjustment costs, firms use advertising expenditures to smooth their payouts, and hence the brand capital investment rate inherits the high volatility of the aggregate and firm-specific shocks.

Eliminating the brand capital adjustment costs also substantially reduces the spreads of the alternative brand capital portfolios, and hence deteriorates the fit of the model on the asset pricing dimension as well. Without brand capital adjustment costs, the spread in the IBK portfolios is significantly reduced (−1.8% here versus −2.9% in the baseline model and −5.2% in the data). The exact same pattern holds for the BKL portfolios. Similarly, the value premium is essentially eliminated.

The positive relationship between the size of brand capital adjustment costs (positive in the baseline model and zero here) and the return spreads/risk (high on the baseline model and small here) can be explained as follows. In production economies, the firm's risk is inversely related to its flexibility to use investment to mitigate the effect of shocks on its effective stream of cash flows to shareholders. The more flexible a firm is in this regard, the less risky it is since the size of the adjustment costs controls how much cash flow management firms can do. Here, without brand capital adjustment costs, all firms are more flexible since they can easily adjust the stock of brand capital. As a result, the overall risk in the economy, and thus the corresponding risk dispersion in the cross section, is significantly reduced.

Turning to the analysis of the effect of financing costs (specification 3), we see that eliminating the equity issuance costs in the model has a small effect on the overall quantity and asset pricing results of the model. The main effect is, as expected, the fact that firms issue equity too often (47% here versus 29% in the baseline model and 25% in the data). Thus, the key conclusions from the model seem to be robust to alternative calibrations of the financing costs.

Finally, we note that the main qualitative results for the SEO regressions across the different calibrations seem stable. The sign and the relative magnitude of the relevant event dummies slope coefficients (Event in IBK regressions, and the three event dummies in Return regressions) is consistent across most specifications, including those reported here. Thus, the ability of the neoclassical investment model to explain the pattern of investment in brand capital (advertising) and stock returns around SEOs is a robust finding that seems to hold across reasonable perturbations of the main calibration of the model.

6. Conclusion

Using a firm-level measure of brand capital stock – a primary form of intangible capital – constructed from advertising expenditures accounting data, we document strong correlations between brand capital and average stock returns in the cross section of US publicly traded firms. We show that a standard neoclassical investment-based model augmented with brand capital, a factor of production that increases firms' operating profits and is subject to adjustment costs, can simultaneously match the asset pricing facts and key properties of the brand capital and physical capital investment rates. For these results to hold, we show that the adjustment costs of brand capital, in addition to adjustment costs of physical capital, have to be sufficiently large. The model also rationalizes the pattern of stock returns and advertising expenditures around seasoned equity offerings identified in previous studies, and which have been interpreted as evidence of limited attention of investors in financial markets. Taken together, our results highlight the importance of brand capital for understanding firms' risk and market value.

Appendix A. Numerical algorithm

To solve the model numerically, we use the value function iteration procedure to solve the firm's maximization problem. The value function and the optimal decision rule are solved on a grid in a discrete state space. We use a multi-grid algorithm in which the number of points is 40 for K and 30 for B. In each iteration we specify a grid of points for physical capital and brand capital, respectively with upper bounds $\bar{K}$ and $\bar{B}$ that are large enough to be non-binding. The grids for physical capital and brand capital stocks are constructed recursively, following McGrattan (1999), that is, $K_i = K_{i-1} + c_{k1}\exp(c_{k2}(i-2))$, where $i = 1, \dots, n$ is the index of grids points and c_{k1} and c_{k2} are two constants chosen to provide the desired number of grid points and two upper bounds $\bar{K}$ and $\bar{B}$, given two pre-specified lower bounds $\underline{K}$ and $\underline{B}$. The advantage of this recursive construction is that more grid points are assigned around $\bar{K}$ and $\bar{B}$, where the value function has most of its curvature.

The state variable x has continuous support in the theoretical model, but it has to be transformed into discrete state space for the numerical implementation. We use a 5 state Markov process for x and a 3 state Markov process for z. In all cases the results are robust to finer grids as well. Once the discrete state space is available, the conditional expectation can be carried out simply as a matrix multiplication. Cubic interpolation is used extensively to obtain optimal physical capital investment and brand capital investment which do not lie directly on the grid points. Finally, we use a simple discrete, global search routine in maximizing the firm's problem.

Appendix B. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.red.2013.05.001.

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