–WEB APPENDIX–

EMPIRICAL ENTRY GAMES WITH COMPLEMENTARITIES:

AN APPLICATION TO THE SHOPPING CENTER INDUSTRY

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WEB APPENDIX A

The Role of the Mall Developer[†]

The purpose of this appendix is to explain how the role of the mall developer is captured by the model in the main text and to show that the current setup can be interpreted as being consistent with a more complex bargaining framework. I also provide more institutional details on the negotiation process that goes on between the mall developers and the anchor stores.

The mall developer is accounted for in the model in three main ways: a) implicitly through the choice of the mall location (and corresponding demographic characteristics), b) explicitly through the inclusion of a mall developer fixed effect in the stores' profit functions and c) through his role as a "focal arbitrator" in the selection of an equilibrium.

To make more clear each of these different facets of the role of the mall developer, I start by specifying the timing of the decisions related with the development of a shopping center:

Phase 1: Mall developer chooses the location of the mall

Phase 2: Developer meets with potential anchor stores and presents them the demographics of the mall and, after negotiations with the developer, anchor stores decide whether or not to enter

Phase 3: Mall is built and smaller retailers make entry decisions.

The first way through which the mall developer influences the character of the mall occurs in Phase 1. By choosing a location (with associated demographics), the mall developer plays an important role in influencing stores' decisions to enter. For example, if the mall developer chooses a location with lower purchasing power, the incentives for high end department stores to enter will be low. This is reflected through the parameters associated with the demographics for this type of anchor stores.

[†]I am especially grateful to Upender Subramanian and Alon Eizenberg for very helpful discussions related with the material presented in this appendix.

In the stage-2 subgame, which is the focus of my paper, the mall location and demographics are taken as given and enter as a set of parameters into each store's profit functions. Rents and allowances given to the anchor stores are negotiated at this stage.

Not much is known about the exact form of the negotiation process used in reality, how much it varies across developer-anchor pairs or across markets, or the extent of asymmetric information between mall developers and anchor stores. Interviews with mall developers and anchor stores representatives who are involved in contractual negotiations suggest that, more often than not, 1) mall developers approach many potential anchors at the same time (as opposed to one at a time, for example), 2) anchor stores are offered low rents and money (e.g. in the form of an allowance for construction) and 3) anchor stores have the final decision rights over whether to agree to contracts. Each of these features is consistent with the theoretical model presented in the main text and discussed here.

The following paragraph from the *Shopping Center Development Handbook* gives an idea of how complex the terms negotiated between the developer and the anchor stores may be: "Department stores in a regional shopping center are treated differently from other tenants. Key department stores may build their own stores on land bought or leased from the developer. They usually do not build their own parking areas, but they should contribute funds to the developer. The developer must have satisfactory reciprocal operating agreements that provide for handling on-site and off-site construction costs, easements, operation of common areas, operating hours, security, the marketing fund or merchants' association, the common "mall/department store" wall and other expenses. Long-term cross-easements and agreements are extremely important to the permanent lender, the tenants and the developer.".

Given the above, and while it is infeasible to model explicitly the bargaining process that goes on between the mall developer and the anchor stores, I can take one step towards showing how the strategic role of the mall developer is accounted for in the model the main text through my choice of specification for the anchor stores' profit functions. More formally, and using a simplified version of the model in the main text, let there be n potential department stores indexed by $i = 1, \ldots, n$ in market m. The profits earned by store i upon entry are given by

$$\hat{\Pi}_i = f(x_m) + g(a_{-i}) + \epsilon_i(a_i = 1), \tag{1}$$

where $f(x_m)$ is a function of the demographic characteristics of market m, $g(a_{-i})$ is a function of the strategies for all stores excluding store i, and ϵ_i is an idiosyncratic term which is private information to store i.

Let the mall developer be D. Now assume that, from D's perspective, entry by store i is worth $v_i \in \mathbb{R}$. For instance, i's entry could lead to additional traffic and therefore business worth v_i for the small stores (not being modeled) that must pay D to set up their store in the mall. Based on some bilateral bargaining, D captures a certain fraction $\lambda \in (0, 1]$ of this value generated for the small stores. Thus, the objective for D is to maximize the business generated for smaller stores by the entry of the anchor stores. We can interpret v_i as a measure of "anchor store fit" for the mall developer. It indicates how much the mall operator values the entrance of store i. This "fit" will induce a set of store-specific subsidies that will influence an anchor store's decision to enter the mall.

To motivate stores to enter, D can provide subsidies or other side-benefits.

Thus, the effective payoff $\tilde{\Pi}_i$ of store i is given by

$$\tilde{\Pi}_i = \hat{\Pi}_i + (1 - \lambda_i) \cdot v_i, \tag{2}$$

and the probability p_i of store i's entry is given by,

$$p_i(a_i = 1) = Pr(\tilde{\Pi}_i > 0). \tag{3}$$

So, the developer's maximization problem can be written as

$$\max_{\lambda} \sum_{i \in N} \Psi(p_i \cdot (\lambda_i \cdot v_i)) \tag{4}$$

$$\text{subject to } \hat{\Pi}_i + (1 - \lambda_i) \cdot v_i > 0, \tag{5}$$

where $\lambda = (\lambda_1, \ldots, \lambda_n)$ denotes the vector of subsidies given to the anchor stores and $\Psi(\cdot)$ is some function (unknown to the econometrician).

Two observations follow:

1. Since stores have private information, the mall developer maximizes his profits using a mechanism (or payment rule) that is incentive compatible and that induces voluntary participation, i.e. a mechanism through which (5) is verified (Page 1992, Myerson 1979 and Myerson 1982).

2. I interpret the stores' profit functions in the model in the main text as $\tilde{\Pi}_i$ with the mall developer fixed effects as $(1 - \lambda_i) \cdot v_i$. Since I do not observe the values of the exchanges between the mall developer and the anchor stores, I account for the fit between the mall developer and each department store by making the stores' reduced form profit functions, and hence the equilibrium played, dependent on the mall developers' identities. Some department stores have more bargaining power and tend to get systematically better subsidies than others. In the model, I try to capture this reality by allowing the stores' probabilities of entering to depend on a store fixed effect and on a mall developer fixed effect that is allowed to vary across stores. At the same time, the mall developers' fixed effects capture the fact that mall developers tend to have long-standing relationships with certain anchors and also that certain developers tend to specialize in specific types of malls. For instance, a developer that specializes in high-end malls is expected to have a positive impact on the entry probabilities of high-end department stores (when compared with discount stores, for example).

Finally, suppose that the constraint in (5) implies multiple equilibria. This leads us to the third way through which the mall developer enters the anchors' entry game. As described in the Web Appendix entitled "MPEC and Multiple Equilibria," I assume that the mall developer acts as a "focal arbitrator" in the selection of an equilibrium.

In sum, the mall developer is present in the model implicitly through the choice of the mall location (and corresponding demographic characteristics), explicitly through the inclusion of a mall developer fixed effect in the stores' profit functions and through his role as a "focal arbitrator" in the selection of an equilibrium.

WEB APPENDIX B

Number of Potential Entrants: Robustness Checks

In this appendix, I test the robustness of the results to changes in the number of potential entrants. I find that the results in the main text are robust across a reasonable range of the number of potential entrants.

In order to test the sensitivity of the results to different numbers of potential entrants I re-estimated the model while allowing for 2, 3 and 10 potential entrants of the "other" category for each of the three tiers of department stores (discount, mid-scale and upscale). To do this exercise we have to assume that any potential "other" entrants of a given tier of are identical (i.e. homogenous with respect to their profit functions).

In order to keep the number of tables manageable, I present only the results from changing the number of "other" potential entrants for the mid-scale tier. The pattern of the results for the other tiers is the same.

I find that, when compared to the model in which there is only 1 potential entrant for the "other mid-scale" category, the intercepts for the firms of type "other mid-scale" become more negative as we add more potential entrants (see panels A of Tables 2, 3 and 4 at the end of this appendix). The result is that adding more potential entrants leads to lower entry probabilities for each of the "other mid-scale" potential entrants. Such change in probabilities is proportional to the change in the number of entrants as we can see from Table 1 (numbers in bold).

The sign and magnitude of all other coefficients (including the strategic effects) remains essentially the same. The full results from the estimation with different number of potential entrants can be seen in Tables 2, 3 and 4.

Table 1
Average ratios of fitted probabilities

The table illustrates the changes in the fitted probabilities for the case in which we allow for 2, 3 and 10 potential entrants of mid-scale department stores that belong to the "other" category.
The numbers in this table were obtained by calculating the ratios of the fitted probabilities (model with one potential entrant on the numerator and with 2, 3 or 10 potential entrants in the denominator) for each market and then averaging across markets.

		Discount Dep. Stores			Mid-scale Dep. Stores			Upscale Dep. Stores		
		Sears	Target	Other Disc	JCP	Mervyn's	Other Mid	Dillards	Macy's	Other Up
Number of	2	1.0030	0.9942	1.0099	0.9991	1.0104	**1.9875**	0.9955	1.0006	0.9924
Potential	3	1.0034	0.9939	1.0175	1.0002	1.0186	**3.0070**	0.9979	0.9969	0.9909
entrants	10	1.0053	0.9942	1.0176	1.0005	1.0169	**10.0456**	0.9961	0.9958	0.9897

The intuition for why the changes in fitted probabilities are proportional to the number of entrants is simple. As an illustration, consider a case in which the potential entrants are a JCPenney (with subscript "1") and some "other" midscale department store (with subscript "2"). When we go from 1 "other" potential midscale to 2 the expression for the probability of entry for JCPenney changes from (1) to (2).

$$p_1^* = \frac{\exp\left(\alpha_1+\beta_1 X_m+\delta_{21}\cdot p_2^*\right)}{1+\exp\left(\alpha_1+\beta_1 X_m+\delta_{21}\cdot p_2^*\right)} \tag{1}$$

$$p_1^* = \frac{\exp\left(\alpha_1+\beta_1 X_m+2\cdot\delta_{21}\cdot p_2^*\right)}{1+\exp\left(\alpha_1+\beta_1 X_m+2\cdot\delta_{21}\cdot p_2^*\right)} \tag{2}$$

Allowing for another midscale potential entrant is equivalent to dividing the probability of entry of each entrant in two. The end result is that we now may have more potential entrants , each with a smaller probability of entry but the joint effect of those two entrants in JCPenney's profits is still large.

Table 2
Parameter Estimates of the Profit Functions of Each Store
(Case with 2 Potential Entrants of "Other Mid-scale")

Panel A: Profit Function Parameters Associated with Demographics and Store Specific Variables

	Discount Dep. Stores			**Mid-scale Dep. Stores**			**Upscale Dep. Stores**		
Variable	Sears	Target	Other Disc	JCP	Mervyn's	Other Mid	Dillards	Macy's	Other Up
Intercept	-0.69^*	-1.93^*	-2.55^*	-0.63	-2.45^*	-1.01^*	-3.80^*	-2.92^*	-1.42^*
	(0.30)	(0.41)	(0.47)	(0.33)	(0.38)	(0.30)	(0.44)	(0.36)	(0.29)
Pop Size	-0.81^*	0.00	0.27	-1.28^*	0.10	-0.39^*	-0.01	0.38^*	0.61^*
	(0.17)	(0.14)	(0.20)	(0.12)	(0.13)	(0.10)	(0.13)	(0.12)	(0.10)
Age	4.30^*	0.42	11.70^*	7.16^*	-6.42^*	-1.48	-1.41	3.89^*	-4.53^*
	(1.00)	(2.09)	(2.59)	(1.37)	(1.50)	(0.97)	(1.01)	(1.14)	(0.69)
Size HH	5.23^*	1.69	5.28	9.37^*	2.06	-3.67^*	1.40	-0.10	-7.53^*
	(1.32)	(1.98)	(3.20)	(1.53)	(1.38)	(1.08)	(1.23)	(1.09)	(0.88)
Sing Fam House	-0.49	2.10^*	0.32	-1.50^*	4.69^*	1.08^*	-1.40^*	-1.14^*	0.35
	(0.32)	(0.77)	(0.83)	(0.30)	(0.90)	(0.50)	(0.41)	(0.33)	(0.24)
House Med Val	0.11	1.57^*	-1.26^*	-0.60^*	0.76^*	-0.33	-1.04^*	1.72^*	0.40^*
	(0.19)	(0.31)	(0.46)	(0.23)	(0.32)	(0.20)	(0.31)	(0.22)	(0.12)
Parking	0.55	0.16	-0.52	1.63^*	2.51^*	0.28	2.31^*	1.61^*	1.62^*
	(0.32)	(0.46)	(0.39)	(0.25)	(0.41)	(0.25)	(0.32)	(0.33)	(0.24)
Store Sqft	2.01^*	-2.70^*	–	2.20^*	-5.48^*	–	-0.15	-0.46	–
	(0.26)	(1.08)	–	(0.29)	(2.31)	–	(0.32)	(0.36)	–
Dist HQ	0.16	-0.44^*	–	-0.05	-0.48^*	–	-0.75^*	0.52^*	–
	(0.10)	(0.13)	–	(0.10)	(0.15)	–	(0.20)	(0.17)	–
Dist DC	-0.28^*	0.10	–	-0.11^*	-0.13	–	-0.68^*	-0.14	–
	(0.07)	(0.11)	–	(0.05)	(0.15)	–	(0.10)	(0.07)	–

Panel B: Profit Function Parameters Associated with Dummy variables

	Discount Dep. Stores			**Mid-scale Dep. Stores**			**Upscale Dep. Stores**		
Variable	Sears	Target	Other Disc	JCP	Mervyn's	Other Mid	Dillards	Macy's	Other Up
Date 2	0.19	-0.06	0.19	0.85^*	0.19	0.09	0.90^*	-0.34	0.29^*
	(0.16)	(0.24)	(0.37)	(0.16)	(0.27)	(0.15)	(0.20)	(0.18)	(0.10)
Date 3	0.51^*	0.30	-0.16	1.28^*	0.94^*	0.32	1.57^*	0.06	0.56^*
	(0.19)	(0.26)	(0.45)	(0.20)	(0.28)	(0.18)	(0.21)	(0.20)	(0.13)
Developer GGP	0.92^*	-0.16	-0.01	1.34^*	0.53	-0.17	0.66^*	1.02^*	0.33^*
	(0.23)	(0.24)	(0.46)	(0.20)	(0.31)	(0.21)	(0.18)	(0.23)	(0.15)
Developer SPG	1.01^*	-0.79	-2.69	1.25^*	-0.01	0.39^*	1.06^*	1.86^*	0.82^*
	(0.31)	(0.42)	(2.78)	(0.21)	(0.37)	(0.19)	(0.20)	(0.22)	(0.15)
Other Dev Medium	0.78^*	-0.81^*	-0.91^*	1.70^*	0.31	0.50^*	0.59^*	0.53^*	0.81^*
	(0.30)	(0.28)	(0.38)	(0.20)	(0.21)	(0.16)	(0.21)	(0.22)	(0.14)

Panel C: Strategic Effects

	Discount	Mid-Scale	Upscale
Discount	−0.46	−0.75	0.92*
	(0.76)	(0.44)	(0.41)
Mid-Scale	0.56	0.55	0.41
	(0.39)	(0.29)	(0.25)
Upscale	−0.41	−1.08*	−0.77*
	(0.33)	(0.24)	(0.20)

Table 3
Parameter Estimates of the Profit Functions of Each Store
(Case with 3 Potential Entrants of "Other Mid-scale")

Panel A: Profit Function Parameters Associated with Demographics and Store Specific Variables

	Discount Dep. Stores			**Mid-scale Dep. Stores**			**Upscale Dep. Stores**		
Variable	Sears	Target	Other Disc	JCP	Mervyn's	Other Mid	Dillards	Macy's	Other Up
Intercept	−0.83*	−2.01*	−2.61*	−0.61	−2.42*	−1.46*	−3.8*	−2.9*	−1.42*
	(0.30)	(0.41)	(0.47)	(0.34)	(0.40)	(0.30)	(0.44)	(0.35)	(0.29)
Pop Size	−0.78*	0	0.27	−1.27*	0.11	−0.34*	−0.02	0.37*	0.59*
	(0.17)	(0.14)	(0.20)	(0.12)	(0.13)	(0.10)	(0.13)	(0.12)	(0.10)
Age	4.45*	0.89	12.01*	7.03*	−6.54*	−1.84	−1.47	3.79*	−4.54*
	(0.99)	(2.10)	(2.60)	(1.39)	(1.50)	(1.09)	(1.03)	(1.16)	(0.70)
Size HH	5.18*	2	5.33	9.24*	1.79	−3.74*	1.38	−0.07	−7.48*
	(1.31)	(1.98)	(3.21)	(1.53)	(1.38)	(1.19)	(1.24)	(1.10)	(0.88)
Sing Fam House	−0.42	2.1*	0.37	−1.53*	4.68*	1.01	−1.4*	−1.15*	0.32
	(0.33)	(0.76)	(0.83)	(0.30)	(0.90)	(0.51)	(0.42)	(0.33)	(0.24)
House Med Val	0.1	1.51*	−1.24*	−0.6*	0.76*	−0.31	−1.04*	1.71*	0.39*
	(0.19)	(0.31)	(0.46)	(0.23)	(0.32)	(0.22)	(0.31)	(0.22)	(0.13)
Parking	0.35	0.02	−0.65	1.71*	2.59*	0.45	2.31*	1.59*	1.62*
	(0.33)	(0.47)	(0.39)	(0.26)	(0.41)	(0.28)	(0.32)	(0.34)	(0.24)
Store Sqft	1.96*	−2.63*	–	2.16*	−5.7*	–	−0.16	−0.44	–
	(0.26)	(1.08)	–	(0.29)	(2.31)	–	(0.32)	(0.36)	–
Dist HQ	0.15	−0.45*	–	−0.06	−0.47*	–	−0.76*	0.52*	–
	(0.10)	(0.13)	–	(0.10)	(0.15)	–	(0.20)	(0.17)	–
Dist DC	−0.28*	0.11	–	−0.11*	−0.14	–	−0.67*	−0.13	–
	(0.07)	(0.11)	–	(0.05)	(0.15)	–	(0.10)	(0.07)	–

Panel B: Profit Function Parameters Associated with Dummy variables

Variable	**Discount Dep. Stores**			**Mid-scale Dep. Stores**			**Upscale Dep. Stores**		
	Sears	Target	Other Disc	JCP	Mervyn's	Other Mid	Dillards	Macy's	Other Up
Date 2	0.15	−0.10	0.15	0.87*	0.20	0.12	0.90*	−0.34	0.29*
	(0.16)	(0.24)	(0.37)	(0.16)	(0.27)	(0.16)	(0.20)	(0.18)	(0.10)
Date 3	0.44*	0.24	−0.21	1.33*	0.99*	0.35	1.58*	0.07	0.58*
	(0.19)	(0.26)	(0.45)	(0.20)	(0.28)	(0.19)	(0.21)	(0.20)	(0.13)
Developer GGP	0.84*	−0.21	−0.05	1.36*	0.55	−0.07	0.66*	1.02*	0.33*
	(0.23)	(0.24)	(0.46)	(0.20)	(0.31)	(0.22)	(0.18)	(0.24)	(0.15)
Developer SPG	0.86*	−0.89*	−2.82	1.31*	0.06	0.49*	1.08*	1.85*	0.81*
	(0.32)	(0.42)	(2.79)	(0.22)	(0.38)	(0.21)	(0.20)	(0.22)	(0.15)
Other Dev Medium	0.66*	−0.88*	−0.99*	1.73*	0.36	0.54*	0.61*	0.53*	0.82*
	(0.30)	(0.28)	(0.38)	(0.20)	(0.20)	(0.17)	(0.22)	(0.23)	(0.14)

Panel C: Strategic Effects

	Discount	Mid-Scale	Upscale
Discount	−0.66	−0.66	0.97*
	(0.76)	(0.45)	(0.41)
Mid-Scale	0.67	0.50	0.35
	(0.39)	(0.29)	(0.26)
Upscale	−0.20	−1.22*	−0.78*
	(0.33)	(0.26)	(0.21)

Table 4
Parameter Estimates of the Profit Functions of Each Store
(Case with 10 Potential Entrants of "Other Mid-scale")

Panel A: Profit Function Parameters Associated with Demographics and Store Specific Variables

Variable	**Discount Dep. Stores**			**Mid-scale Dep. Stores**			**Upscale Dep. Stores**		
	Sears	Target	Other Disc	JCP	Mervyn's	Other Mid	Dillards	Macy's	Other Up
Intercept	−0.93*	−2.09*	−2.70*	−0.65	−2.45*	−2.78*	−3.77*	−2.87*	−1.39*
	(0.17)	(0.14)	(0.19)	(0.13)	(0.13)	(0.10)	(0.12)	(0.12)	(0.10)
Age	4.52*	1.08	12.09*	6.86*	−6.62*	−1.87	−1.49	3.73*	−4.57*
	(0.97)	(2.07)	(2.55)	(1.37)	(1.59)	(1.14)	(1.01)	(1.14)	(0.73)
Size HH	5.10*	2.05	5.27	9.13*	1.68	−3.58*	1.41	0.01	−7.39*
	(1.29)	(1.99)	(3.02)	(1.50)	(1.42)	(1.24)	(1.20)	(1.07)	(0.91)
Sing Fam House	−0.38	2.10*	0.38	−1.53*	4.68*	0.94	−1.41*	−1.17*	0.30
	(0.31)	(0.74)	(0.81)	(0.32)	(0.86)	(0.49)	(0.38)	(0.33)	(0.23)
House Med Val	0.10	1.49*	−1.22*	−0.61*	0.75*	−0.28	−1.04*	1.69*	0.38*
	(0.20)	(0.31)	(0.46)	(0.23)	(0.33)	(0.21)	(0.31)	(0.22)	(0.12)
Parking	0.23	−0.06	−0.73	1.72*	2.56*	0.47	2.31*	1.61*	1.64*
	(0.31)	(0.47)	(0.40)	(0.28)	(0.44)	(0.30)	(0.32)	(0.36)	(0.24)
Store Sqft	1.97*	−2.59*	–	2.10*	−5.93*	–	−0.16	−0.44	–
	(0.27)	(1.07)	–	(0.28)	(2.35)	–	(0.33)	(0.36)	–
Dist HQ	0.15	−0.46*	–	−0.05	−0.46*	–	−0.76*	0.52*	–
	(0.10)	(0.13)	–	(0.09)	(0.15)	–	(0.20)	(0.16)	–
Dist DC	−0.27*	0.12	–	−0.12*	−0.15	–	−0.67*	−0.14	–
	(0.07)	(0.11)	–	(0.05)	(0.14)	–	(0.10)	(0.07)	–

Panel B: Profit Function Parameters Associated with Dummy variables

Variable	**Discount Dep. Stores**			**Mid-scale Dep. Stores**			**Upscale Dep. Stores**		
	Sears	Target	Other Disc	JCP	Mervyn's	Other Mid	Dillards	Macy's	Other Up
Date 2	0.11	−0.12	0.11	0.87*	0.20	0.12	0.91*	−0.33	0.29*
	(0.15)	(0.25)	(0.36)	(0.16)	(0.26)	(0.15)	(0.20)	(0.19)	(0.11)
Date 3	0.39*	0.20	−0.25	1.31*	0.99*	0.35	1.59*	0.08	0.59*
	(0.18)	(0.25)	(0.44)	(0.21)	(0.27)	(0.18)	(0.21)	(0.20)	(0.13)
Developer GGP	0.81*	−0.24	−0.08	1.34*	0.54	−0.03	0.66*	1.01*	0.32*
	(0.21)	(0.26)	(0.48)	(0.21)	(0.30)	(0.20)	(0.18)	(0.24)	(0.15)
Developer SPG	0.80*	−0.94*	−2.86	1.29*	0.07	0.48*	1.08*	1.86*	0.82*
	(0.30)	(0.42)	(2.99)	(0.23)	(0.39)	(0.22)	(0.21)	(0.23)	(0.15)
Other Dev Medium	0.60*	−0.93*	−1.04*	1.72*	0.36	0.50*	0.63*	0.54*	0.83*
	(0.29)	(0.29)	(0.43)	(0.20)	(0.22)	(0.16)	(0.21)	(0.23)	(0.15)

Panel C: Strategic Effects

	Discount	Mid-Scale	Upscale
Discount	-0.73	-0.59	1.00^{*}
	(0.76)	(0.44)	(0.42)
Mid-Scale	0.76	0.46	0.30
	(0.39)	(0.27)	(0.27)
Upscale	-0.09	-1.19^{*}	-0.80^{*}
	(0.33)	(0.27)	(0.22)

WEB APPENDIX C

Identification Issues in Games of Complete Information: A Review

In this Web Appendix, I review briefly the identification issues encountered in complete information games and the approaches that have been used to deal with them.

Identification Issues in Entry Games of Complete Information

If players possess full information about other players' payoffs and only pure strategies[1] are played, players' optimal strategies can be represented by a simultaneous discrete response system of the type studied by Heckman (1978). An example of such system can be the following model with two firms

$$\Pi_1 = x\beta_1 + \delta_{21} \cdot y_2 + \epsilon_1$$

$$\Pi_2 = x\beta_2 + \delta_{12} \cdot y_1 + \epsilon_2$$

$$y_i = \begin{cases} 1 & \text{if} \quad \Pi_i > 0 \\ & \qquad\qquad \text{for } i = 1, 2. \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where the set of endogenous variables Y consists of $Y = \{(0,0),(0,1),(1,0),(1,1)\}$, which in our case correspond to the entry decisions of the two firms.

In this type of system (i.e. a simultaneous-equation model with underlying discrete latent variables), a well-defined likelihood function for the set of observable outcomes exists if and only if the mapping from equilibria to inequalities on profits is a well-defined function. This means that, in order to have a well-defined likelihood function, the firms' equilibrium strategies (which depend

[1] Aradillas-Lopez (2005) has shown that if mixed-strategies are allowed, a well-defined likelihood function exists for all the outcomes of the game under conditions much weaker than when only pure-strategies are allowed. Anyway, in this case, identification cannot still be achieved without any additional assumptions.

on the error terms, the market observables and the model parameters) have to exist and be unique.

When a game has multiple equilibria, there is no longer a unique relation between players' observed strategies and those predicted by the model. As shown in Bresnahan and Reiss (1991), this always happens in simultaneous-move Nash models when the errors have sufficiently wide supports.

Amemiya (1974), Heckman (1978) and others have shown that the only way to get a well-defined likelihood function in this type of model is by imposing a "coherency condition". Such condition seems undesirable in an entry model since it does not preserve the simultaneous-interaction element of the model. In order to avoid the coherency condition, Bresnahan and Reiss (1990), Bresnahan and Reiss (1991) impose the constraint that entry of an additional firm is always costly (i.e. it always reduces the other firms' profits) and redefine the space of outcomes (by aggregating nonunique equilibrium outcomes) in such a way that the game is transformed into one that predicts unique equilibria.[2]

The disadvantage of these approaches is that they result in efficiency losses and limit the ability of the econometrician to make predictions over the entire set of observable outcomes.

Another way of dealing with the multiplicity of equilibria issue is to develop some theory of equilibrium selection. For example, one can make an assumption on the order in which the players move (as in Berry 1992 and Mazzeo 2002)[3] or model more formally (i.e. parametrically) an equilibrium selection mechanism (as in Bajari, Hong, and Ryan 2010). The use of an appropriate equilibrium selection rule assures the existence of a well-defined likelihood function for the entire space of observable outcomes . The problem with this latter approach is that consistency of the estimation depends critically on the validity of the assumed selection rule which is not testable.

[2]Note that if there is firm heterogeneity, even imposing these conditions, we can have multiple equilibria with different number of firms.

[3]Actually, imposing an order of entry may not be enough to guarantee uniqueness. For example, Mazzeo (2002) rules out payoffs increasing in the number of competitors and imposes that same-type competitors must affect profits at least as much as different-type competitors.

WEB APPENDIX D

MPEC and Multiple Equilibria

In this appendix, I illustrate, with an example, the benefits of using the MPEC (Mathematical Programming with Equilibrium Constraints) approach to estimate games with multiple equilibria. Here I entertain a simplified version of the entry model discussed in the paper and assume that there are only two firms/players (with different "types") that have to decide whether or not to enter in a series of markets based on a single observed market characteristic (X) and on their beliefs with respect to the action to be taken by the other firm. For this setup, the ex-post profits earned by store i ($i = 1, 2$) if the firm decides to enter in a specific market (i.e. $a_i = 1$) are given by

$$\Pi_i \left(X_m; \theta \right) = \alpha_i + \beta_i X_m + \delta_{ji} 1 \left\{ a_j = 1 \right\} + \epsilon_i \text{ for } i \neq j \tag{1}$$

where α_i is the store-specific mean profitability level, β_i measures the impact of the market characteristic X on store i's profits and δ_{ji} measures the impact of store j's entry on store i's profits.

Under the same distributional and informational assumptions used in the main text, the system of equations that defines the equilibrium for a specific market m can be written as:

$$\begin{cases} p_1^* = \frac{\exp\left(\alpha_1 + \beta_1 X_m + \delta_{21} \cdot p_2^*\right)}{1 + \exp\left(\alpha_1 + \beta_1 X_m + \delta_{21} \cdot p_2^*\right)} \\ \\ p_2^* = \frac{\exp\left(\alpha_2 + \beta_2 X_m + \delta_{12} \cdot p_1^*\right)}{1 + \exp\left(\alpha_2 + \beta_2 X_m + \delta_{12} \cdot p_1^*\right)} \end{cases} \tag{2}$$

where p_1^* and p_2^* are the firms' equilibrium entry probabilities.

For any (X_m, $\theta = \{\alpha, \beta, \delta_{12}, \delta_{21}\}$), the game has multiple equilibria if more than one set of values $\{p_1^*, p_2^*\}$ satisfies (2).

As shown in the *Model Identification* section in the main text, the firms' profit function structural parameters can be estimated by observing the entry decisions of these two firms for a cross-section of M markets for which we also observe the value of the market characteristic X. So, and for a given set of markets, the maximum likelihood estimator, when using the MPEC approach, is the solution to

$$\max_{(\theta,p_1^*,p_2^*)} L(X_m;\theta) \quad = \quad \prod_{m=1}^{M}\prod_{i=1}^{2}\left(p_i^*(X_m;\theta)\right)^{1\{a_i=1\}} \cdot \left(1-p_i^*(X_m;\theta)\right)^{1\{a_i=0\}}$$

subject to

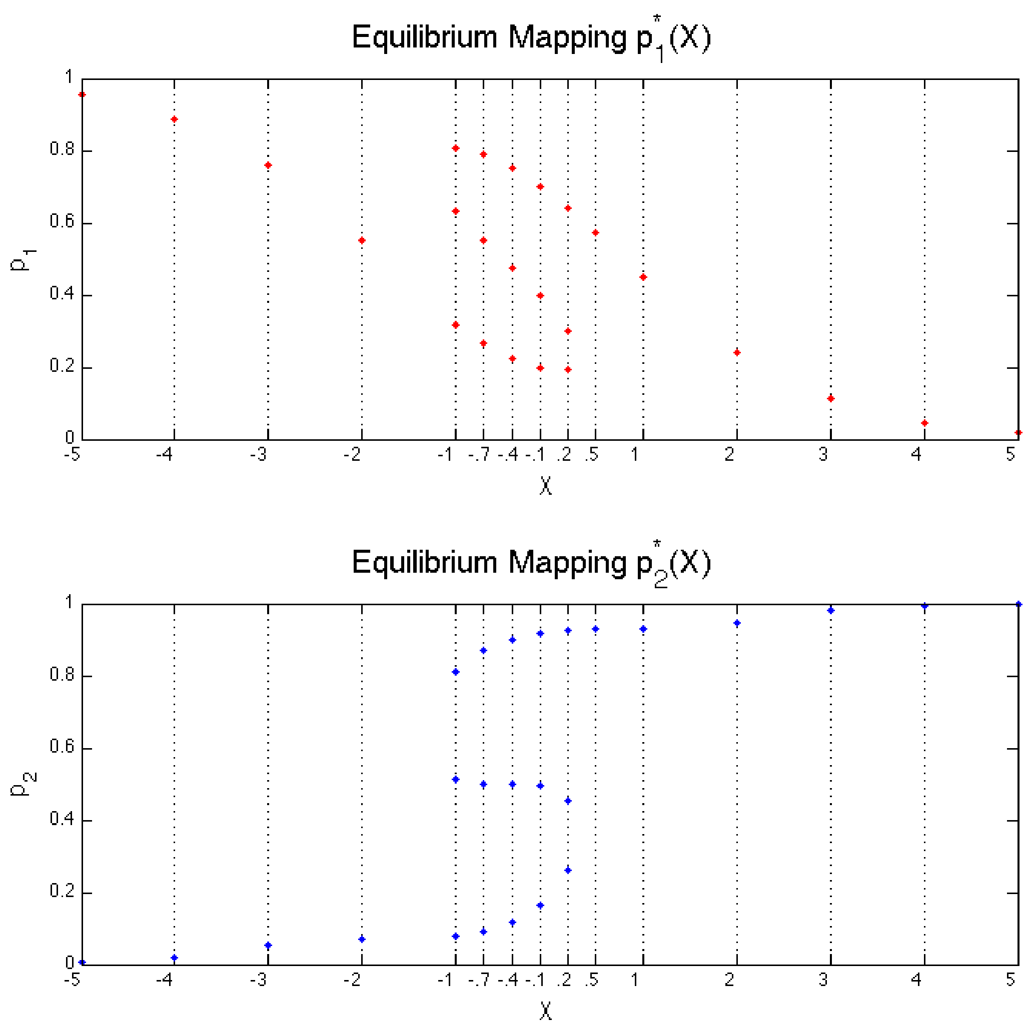
$$\begin{cases} p_1^* = \frac{\exp\left(\alpha_1+\beta_1 X_m+\delta_{21}\cdot p_2^*\right)}{1+\exp\left(\alpha_1+\beta_1 X_m+\delta_{21}\cdot p_2^*\right)} \\ \\ p_2^* = \frac{\exp\left(\alpha_2+\beta_2 X_m+\delta_{12}\cdot p_1^*\right)}{1+\exp\left(\alpha_2+\beta_2 X_m+\delta_{12}\cdot p_1^*\right)} \end{cases} \tag{3}$$

Let $P^*(\theta)$ be the set of all $p^* = \{p_1^*, p_2^*\}$ such that the constraint in (3) is satisfied. Estimating a model such as the one above using a standard fixed-point algorithm approach requires that, for each θ, one finds all the $p^* \in P^*(\theta)$ that solve the system in (3), computes the likelihood at each $p^* \in P^*(\theta)$ and reports the maximum. Finding all equilibria is an intractable problem, especially in more complex games. Also, the resulting likelihood function will often be non-smooth or even discontinuous if there are multiple equilibria, which creates difficulties for the optimization procedure. In contrast, multiplicity of equilibria will not create discontinuities or lack of differentiability in the MPEC approach.

For the purpose of illustration, the true values of the structural parameters were specified to be $\alpha_1 = -2$, $\alpha_2 = -3$, $\beta_1 = -1$, $\beta_2 = 2$, $\delta_{12} = 8$ and $\delta_{21} = 3$. I further assume that $X_m \in \{-5,-4,-3,-2,-1,-0.7,-0.4,-0.1,0.2,0.5,1,2,3,4,5\}$. Figure 1 shows the equilibrium mappings for each player for different values of X.

For the cases in which $X_m \in \{-5,-4,-3,-2,1,2,3,4,5\}$ we don't have multiple equilibria in this game. But, if $X_m \in \{-1,-0.7,-0.4,-0.1,0.2,0.5\}$ we have multiple equilibria. For example, if $X_m = -1$ we can verify that there is an odd number (3) of isolated equilibrium points:

Figure 1



$$(p_1^I, p_2^I) = (0.318, 0.0790)$$

$$(p_1^{II}, p_2^{II}) = (0.6313, 0.5126)$$

$$(p_1^{III}, p_2^{III}) = (0.8078, 0.8119)$$

Essentially, we have an equilibrium (III) in which both firms enter with high probability, another in which they both enter with low probability (I) and a third one (II).These equilibria are a result of the fact that, in this example, $\delta > 0$ (i.e. both firms benefit from each other's entry). For certain values of X, each firm is willing to enter only if it believes the other firm will also enter (because of the positive spillovers).

To generate artificial data based on the true values for θ and X, I assume that the same equilibrium is played whenever firms face a market with characteristic X (i.e. there is some mechanism by which the players always comply to a specific equilibrium when facing the same market characteristic).[1]

Suppose we pick equilibrium I (when $X = -1$) to generate the synthetic data . The generated outcomes for 1000 markets with $X = -1$ will follow a pattern of the type in Table 1. These simulated outcomes are consistent with the fact that, whenever $X = -1$, the *most likely* outcome is that both firms choose not to enter.

[1]This assumption is required for estimation an identification purposes (as discussed in the *Model Identification* section of the paper). This is a common assumption in the context of incomplete information games (independently of the method of estimation used, MPEC or other) and while it is typically formulated in this way (i.e. that the same equilibrium is always played in observationally equivalent markets) this "observationally equivalent" is usually interpreted in a loose sense (i.e. that firms play the same equilibrium at the neighborhood of X). For more on the rationale for this assumption please refer to the sub-section *The mall developer's role as a "focal arbitrator"* at the end of this Appendix.

Table 1

Firm's 1 decision	Firm's 2 decision		Total
	Not enter	Enter	
Not enter	636	33	669
Enter	304	27	331
Total	940	60	1,000

To recover the true parameters based on this data, a NXFP (Nested Fixed Point) approach would require that, for each value of θ, all the possible p^* are calculated and then the one that maximizes the likelihood is picked before moving to the next tentative θ. This would need to be done in order to try to delineate the likelihood function to be maximized. In contrast, in MPEC, the objective function depends explicitly on both θ and $p^* = (p_1^*, p_2^*)$, which means that both θ and p^* are taken as free parameters to be estimated (in NXFP, only θ is free).

The MPEC/constrained optimization approach works better since we don't need to solve all the equilibria; instead, since the objective and constraints' functions are all smooth, the gradient/derivative information in the joint parameter-equilibria $(\theta, (p_1^*, p_2^*))$ space can be used to find an ascent direction for MLE. The idea is that, if from the gradient/derivative information, the MLE objective function will be decreasing from this equilibrium to another equilibrium or all other equilibria, then there is no need to jump to that other equilibrium to find out that the MLE objective is indeed lower. This means that iterates can move through infeasible regions and feasibility only needs to be satisfied at the solution, which makes the process much faster.

The mall developer's role as a "focal arbitrator"

In the main text it is said that "since the game may have many possible equilibria, it is assumed that the mall planner chooses the equilibrium that maximizes some criteria." This is equivalent to interpreting the mall developer as a "focal arbitrator".

Schelling's focal-point effect theory (Schelling 1960) postulates that, in games with multiple equilibria, if there is one focal equilibrium then one should expect to observe that equilibrium. There are many ways that can make players to become focused on one equilibrium. One of them is by the presence of a "focal arbitrator," an individual that can determine the focal equilibrium in a game by publicly suggesting to the players that they should all implement this equilibrium. In many games, it may be hard to determine who plays the role of the "focal arbitrator," and hence to justify the assumption required for identification that markets with the same characteristics have the same equilibrium. But, in the current application it seems plausible to assume that this role is played by the mall developer. Thus, a game with a large set of equilibria is a game in which the mall developer can substantially influence the stores' behavior.

In this context, the mall developer is assumed to choose the equilibrium that maximizes some criteria. These criteria may be in general related with the developer's payoff function which may vary across mall developers.

The approach of having an "arbitrator" with some unobserved payoff function is superior to an alternative, for example, where one would have to specify an explicit equilibrium selection rule as a function of the mall developer's payoff function. By addressing the multiple equilibria issue in this way, we circumvent the problem of having to impose such restrictions and functional forms at the same time that we provide a rationale for why the same equilibrium is played in markets with similar characteristics. The model parameters are then estimated such that the equilibrium selected is the one that is most likely to be played given the data. Lastly, this interpretation is consistent with both anchors and the mall developer having some bargaining power; as opposed to a situation in which the observed outcomes are solely the result of the mall developer's or the anchors' decisions.

WEB APPENDIX E

Brief comparison of MPEC with alternative estimation methods

In this appendix, I briefly compare the MPEC (Mathematical Programming with Equilibrium Constraints) approach with other methods that have been proposed in the literature for estimating discrete games of incomplete information.

The standard nested fixed-point algorithms, like the ones used by Seim (2006), Orhun (2005), Zhu and Singh (2009) and Zhu, Singh, and Manuszak (2009), require the repeated solution of the model for each trial value of the parameters to be estimated. Explicitly solving for the equilibrium may be computationally burdensome for complex models. Furthermore, the (possible) multiplicity of fixed points may render this method impractical.

To address some of the issues associated with nested-fixed point methods, a two-step pseudo maximum likelihood (PML) approach (see, for example, Aguirregabiria and Mira 2002; Bajari et al. 2010; Ellickson and Misra 2008) has been proposed. In this approach, instead of solving the system of beliefs in (13) for each parameter vector, the researcher assumes that, in the data, only one of possible many equilibria is actually being played by the agents. So, by observing the entry behavior across a large number of markets, the econometrician can consistently estimate the conditional choice probabilities $\hat{p}_i(a_i = 1|x_m, T)$ that appear on the right hand side of equation (13), using non-parametric methods and looking at the firms' observed choices. These conditional choice probabilities are then plugged into a pseudo-likelihood function and the parameters are determined by standard maximization. If the pseudo-likelihood function is based on a consistent nonparametric estimator of the firms' beliefs, the two-step estimator is consistent and asymptotically normal.

The PML estimator is computationally simple but has several important limitations. First, this estimator requires a non-parametric first-stage which is infeasible in most empirical contexts. For instance, the shopping center game I consider has over half a dozen market characteristics. The first stage would then require estimating a non-parametric entry strategy function for each firm in over half a dozen dimensions. Second, the nonparametric estimator obtained in the first-step can be very imprecise in small samples generating serious biases in the estimated structural parameters. Last, the PML estimator is a limited information estimator. Thus, it is less efficient than a full information method where all the parameters are estimated simultaneously.

Another method is the nested pseudo likelihood (NPL) method proposed by Aguirregabiria and Mira (2007) which is basically a recursive extension of the two-step PML estimator. The NPL estimator is more efficient than the two-step estimators and does not require that one starts the algorithm with a consistent estimator of the choice probabilities. However, this method also has some important limitations. First, since the iterative procedure is performed using a method of successive approximations, this method is not sure to converge to the equilibrium that generated the data when this equilibrium is *unstable*. Also, to deal with multiple equilibria issues, Aguirregabiria and Mira (2007) claim that the researcher should initiate the NPL algorithm with different guesses of the firms' choice probabilities such that, if different fixed-points are obtained, the one which maximizes the likelihood function is chosen. Unfortunately, this assumes that, if there are multiple fixed-points, the researcher is able to recover all of them, which may not be possible altogether.

Another solution to estimate this type of models is to focus on set identification using a "bounds" approach as in Tamer and Ciliberto 2004; Andrews, Berry, and Jia 2005. The basic idea of this approach is that, while the model may not make exact predictions about outcomes, it does still restrict the range of multiple outcomes. Although the case of multiple equilibria seems to fit this case nicely, these methods still raise a few important econometric questions such as how to prove consistency of the estimators and how to place confidence regions on the set of parameters that satisfy the model.

Conceptually, the MPEC approach that I follow is not more complicated than the above methods. By incorporating the constraints directly in the maximization problem, the researcher can proceed in only one step which makes this method capable of producing efficient estimates (as opposed to the two-step methods mentioned above). The MPEC estimator can be shown to be consistent, asymptotically normal and efficient.